An 'Austrian' Model of International Specialization*

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March 2026

Abstract

We develop a general equilibrium model of international trade in which the temporal structure of production is a key determinant of comparative advantage. Building on Böhm-Bawerk's theory of capital, the model formalizes the idea that production processes with longer average periods of production ($\mathcal{APP}$s) entail higher financing costs due to the time lag between input payments and revenue realization. We embed this insight into a multi-sector Ricardian framework with endogenous interest rates. Under autarky, countries with more patient consumers or more developed financial markets exhibit lower equilibrium interest rates and higher wage rates. With international trade, these countries typically gain a comparative advantage in sectors with longer $\mathcal{APP}$s, though the model can also generate multiple equilibria and unconventional specialization patterns. We extend the framework to include trade costs (inclusive of shipment delays), global value chains, and international capital-market integration. Empirically, we present evidence showing that countries with more developed financial systems export disproportionately more in sectors with longer $\mathcal{APP}$s, even after controlling for standard neoclassical and institutional determinants of comparative advantage.

*A previous version of this paper was entitled "An 'Austrian' Model of Global Value Chains." We thank Robert Stehrer for recommending Janek Wasserman's *"The Marginal Revolutionaries: How Austrian Economists Fought the War of Ideas,"* which served as an initial inspiration for the paper. We are grateful to the Editor, three anonymous referees, and to participants at various institutions for their helpful comments. Carl Christian von Weizsäcker and Iván Werning were particularly instrumental in helping us better understand some of our results. Antràs acknowledges financial assistance from the Ferrante Fund at Harvard University. Finally, we wish to humbly dedicate this paper to the memory of the great Ron Findlay, who passed away on October 8th, 2021, and with whom we would have loved to discuss this work.

"The disadvantage connected with the capitalist method of production is its sacrifice of time. The roundabout ways of capital are fruitful but long; they procure us more or better consumption goods, but only at a later period of time. This proposition [...] is one of the ground pillars of the theory of capital." *The Positive Theory of Capital*, Eugen Böhm-Bawerk (1889, p. 82).

1 Introduction

Time is an essential aspect of every production process. From the procurement of raw materials to the design, fabrication, and distribution of goods, each phase unfolds over a specific temporal span shaped by technological and economic considerations. The role of time in production has long been central to economic thought, from the Classical contributions of Ricardo (1817) and Marx (1867) to the more systematic treatment in Böhm-Bawerk's theory of capital (Böhm-Bawerk, 1889). This latter 'Austrian' view of capital – echoed in earlier work by Jevons (1871) and subsequently refined by thinkers such as Wicksell (1934), Hicks (1939), Dorfman (1959), and Weizsäcker (1971) – placed particular emphasis on the relevance of the temporal gap between the timing of payments to productive inputs and the timing of the eventual sale of the goods embodying these inputs.

While the Austrian theory of capital inspired a large theoretical literature in the 20th century, its implications for trade and specialization in a globalized economy remain underexplored. This paper seeks to fill that gap. We develop a micro-founded general equilibrium model of international trade in which the duration of production plays a central role in shaping comparative advantage. In doing so, we build a bridge between the 'Austrian' tradition and modern international trade models.

Our approach begins by revisiting the canonical insight of Böhm-Bawerk: that production is not instantaneous, and that time introduces a wedge between input expenditures and revenue realization. We operationalize this idea through Böhm-Bawerk's concept of the *average period of production* ($\mathcal{APP}$), a measure of the temporal gap between factor payments and the sale of final goods. The $\mathcal{APP}$, closely related to Böhm-Bawerk's and especially Hicks' (1939) original formulations, becomes a central determinant of production costs in the presence of positive interest rates. Firms operating in higher $\mathcal{APP}$ (or *time-intensive*) sectors incur greater financial opportunity costs, as labor expenditures must be financed over longer horizons.

We embed this insight into a multi-sector Ricardian model with a continuum of goods à la Dornbusch et al. (1977), extended to account for the time-phased nature of production. Each sector is characterized by a distinct temporal structure of production, and a sector's unit cost is a function not just of wages and productivity, but also of the temporal profile of its production process and the prevailing interest rate. Put differently, firms not only demand labor, but also financial capital, and differentially so across sectors. On the supply side, we model the capital market using a stylized overlapping generations framework in the spirit of Caballero et al. (2008), where individuals save while young and consume in old age. This structure yields a tractable and micro-founded supply of capital, governed by labor income, the interest rate, the level of financial development, and the propensity of agents to save. Autarkic equilibrium interest rates are determined endogenously via the interaction of aggregate capital demand and the intertemporal preferences of agents.

We then turn to the open-economy implications of the model. When two countries differ in their degree of patience or financial development, the resulting interest rate differential creates asymmetries in sectoral production costs. Countries with lower interest rates face smaller opportunity costs for time-intensive production processes, and thus develop a comparative advantage in sectors with longer $\mathcal{APP}$s. This mechanism generates a novel basis for trade, distinct from traditional endowment- or productivity-based explanations. More formally, our Proposition 3 establishes that countries with lower interest rates specialize in sectors featuring longer $\mathcal{APP}$s, provided that the ranking of $\mathcal{APP}$s exhibits no reversals.

This result parallels but also provides a spin to the Heckscher-Ohlin (H-O) theorem. Whereas the standard H-O framework focuses on factor endowments and assumes homogeneous production timelines, our model emphasizes the endogenous interaction between capital markets and production time. Moreover, in our setting, capital is not a static input but a dynamic one tied to the financing of labor along a temporal production path. This distinction has important implications. For instance, the relationship between a sector's capital intensity and its $\mathcal{APP}$ may be non-monotonic, and in some cases, ranking reversals between time and capital intensity may occur. These phenomena generate outcomes that are not possible in well-behaved neoclassical trade models, including multiple equilibria and unexpected patterns of specialization. These peculiarities arise because, while financial capital is homogeneous and freely mobile across sectors, the physical capital goods it finances are *heterogeneous* within and across sectors due to variation in the temporal structure of production. Nevertheless, we show that the factor-content of trade implications of our framework remain aligned with those of the Heckscher-Ohlin-Vanek model.

To illustrate our results, we present special versions of the model that permit analytical results, and we also explore numerical simulations of the model. A key benchmark is the Wicksellian point-input, point-output model (Wicksell, 1934; Dorfman, 1959; Findlay, 1978), where all labor is used at the start of production, and output is realized after a fixed duration. This stylized setting yields closed-form solutions and sharp comparative statics. We extend the analysis to more general production functions with labor inputs used along the whole production interval and demonstrate that our main comparative advantage results are robust to these generalizations.

We further enrich the model by considering extensions along several dimensions. First, we introduce trade costs, both of the standard iceberg variety and of a temporal nature – such as delays in shipment and payment. These frictions amplify the impact of interest rate differentials by raising the cost of financing exports, especially for countries with high costs of capital. Second, we model global value chains (GVCs) as sequential production processes, where intermediate goods can be traded across borders at different stages. In this setting, our insights on the role of time carry over, as each stage's $\mathcal{APP}$ continues to interact with interest rates to shape the geography of production. Furthermore, we show that a worldwide decline in interest rates fosters an increase in the share of GVC trade in world trade. Third, we consider the consequences of various forms of partial capital-market integration across countries.

When cross-country variation in the supply of financial capital is primarily shaped by financial development, our model delivers a natural link between comparative advantage and institutional quality, reminiscent of Nunn (2007) and Manova (2013), but grounded in cross-sectoral variation in the temporal structure of production rather than contract incompleteness or external capital dependence. Empirically, we test the model's predictions using detailed country-industry-level export data, combined with a recent measure of the $\mathcal{APP}$ developed by Antràs and Tubdenov (2025). Exploiting cross-sectional and time variation in financial development, we find robust evidence that countries with more developed capital markets (and thus presumably lower effective interest rates) tend to export disproportionately more in sectors with longer average production periods ($\mathcal{APP}$s). This relationship holds even when controlling for standard Heckscher-Ohlin variables such as physical and human capital endowments, and for the interaction of financial development with a sector's external capital dependence. Moreover, the interaction between time intensity and financial development exhibits predictive power comparable to, or exceeding, traditional determinants of comparative advantage.

Taken together, our theoretical and empirical findings underscore the importance of the temporal structure of production in shaping global trade patterns. While modern trade theory has made significant strides in incorporating heterogeneity in productivity, demand, and institutional quality, the time dimension of production has remained largely in the background. This paper brings time to the forefront, offering a new lens through which to view comparative advantage and international specialization.

As is evident from our opening paragraph, our framework builds on a long tradition of 'Austrian' models that begins with the descriptive work of Jevons (1871) and Böhm-Bawerk (1889), continues with the early influential formalization in Wicksell (1934), and culminates with the modern treatments in Metzler (1950), Dorfman (1959), Weizsäcker (1971), and Findlay (1978), among others. Our work is particularly related to Findlay (1978), who also considered an 'Austrian' model of international trade. While his model also linked time and interest rates to specialization patterns, it did so under restrictive assumptions. More specifically, Findlay (1978) developed a special two-sector Wicksellian point-input, point-output model in which capital was only used in a single sector (an upstream sector). Findlay (1978) related comparative advantage to the interaction of time intensity and autarkic differences in interest rates, but in focusing on that special model, he fell short of relating his notion of time intensity to the measure of the $\mathcal{APP}$ developed by Böhm-Bawerk (1889) and Hicks (1939). More importantly, Findlay (1978) focused throughout on trade equilibria featuring factor price equalization. By relaxing the assumption of factor price equalization, we derive a richer set of equilibrium predictions, many of which are novel to our paper. Moreover, we show that time-versus-capital intensity ranking reversals can generate complex, and sometimes non-monotonic, specialization patterns not captured in earlier 'Austrian' or neoclassical models.[1]

Another closely related paper to ours is Kim and Shin (2023), who develop a model of sequential production with 'Austrian' features. Their emphasis, however, is on how the optimal number of symmetric stages of production is shaped by interest rates, and how the longer production times associated with cross-border fragmentation shape the optimal extent of offshoring. Furthermore, they show that, in equilibrium, offshoring, inventories, trade and productivity are all decreasing in interest rates.[2] In our framework, the number of goods or stages is fixed, and we instead emphasize cross-sector heterogeneity in time intensity in shaping international specialization.

Our work is also related to previous international trade frameworks featuring heterogeneous capital goods. The neo-Ricardian 'time-phased' model developed by Steedman and Metcalfe (1977) considers heterogeneous circulating capital and positive profits in the Sraffa (1960) tradition. Their goal is to demonstrate that choice-of-technique effects can overturn the main Heckscher-Ohlin theorems: factor-price equalization may fail, while Stolper-Samuelson and Rybczynski results can reverse (see, however, Ethier, 1979).[3] Although some of our unconventional results stem from the heterogeneous nature of capital goods, we show that those results are tied to the so-called *Wicksell effect* rather than to reswitching effects. Also related to our work are applications of vintage-capital theory to international trade. This literature builds on the seminal two-sector putty-clay model in Bardhan (1966), and treats machines of different ages as distinct factors, with comparative advantage being shaped by the technical

[1]In the title of his paper, Findlay (1978) wrote Austrian between quotation marks because the so-called 'Austrian' theory of capital was independently developed by a British economist (Jevons) and because it was turned mainstream by a Swedish economist (Wicksell). We also adopt quotation marks in our title, but in doing so, we are frankly more motivated by the fact that the term *Austrian economics* is now commonly associated with the latest phases of the Austrian school of economics, when Hayek and especially Mises turned this school into a heterodox branch of economics largely focused on advocating libertarian ideals (Wasserman, 2019).

[2]Bruno et al. (2018) use balance sheet proxies for the length of production chains (specifically, inventories, accounts payable and accounts receivable) to unveil a negative impact of a strong dollar (a proxy for a tightening of dollar credit) on production length.

[3]More broadly, the 'anything goes' result in Mas-Colell (1989) indicates the difficulties in performing comparative statics on the steady state of a model with heterogeneous capital goods.

change embodied in these machines. Some of the papers in this literature study departures from the main theorems of neoclassical trade theory (see, for instance, Petith, 1972), but these departures are distinct in nature from those studied in our 'Austrian' framework.

Although an explicit treatment of production and delivery time is rare in international trade, a few pioneering papers, other than those discussed above, deserve discussion. Deardorff (2003) explores the importance of time in international trade by developing a model in which products depreciate over time, perhaps due to demand changes, and producers can choose to produce and trade faster using production techniques that are more physical capital intensive. Evans and Harrigan (2005) and Hummels and Schaur (2013) also develop frameworks in which consumers value speedier delivery times, and they both show that the pattern of specialization and mode of transport appear to be consistent with the notion that relatively *time sensitive* goods are more likely to be shipped from nearby locations – in Evans and Harrigan (2005) – or via faster shipping methods, such as air shipping – in Hummels and Schaur (2013). Relatedly, Djankov et al. (2010) show that shipping delays in exporting countries (due, for instance, to inefficient trade facilitation practices) significantly depress exports and particularly so for time-sensitive goods, such as perishable agricultural products. In our framework, production and delivery times are important, but rather than emphasizing delay costs motivated by changes in demand, we focus on financial capital costs associated with letting production processes 'mature', potentially to improve their productivity. Furthermore, unlike in Deardorff (2003), we do not assume that faster production involves more capital intensive techniques; instead, we show that longer production processes are in fact associated with a relatively higher demand for working capital. Still, in the presence of interest rate differences across countries, the pattern of international specialization is driven by time intensity rather than by capital intensity.[4]

The rest of the paper is organized as follows. Section 2 develops the closed-economy version of the model, introducing the concept of the $\mathcal{APP}$ and its implications for production costs. Section 3 provides additional results under tractable functional form assumptions. Section 4 presents the open-economy setting and derives the pattern of specialization under free trade. Section 5 presents empirical evidence linking country-and-industry export data to the interaction of sectoral measures of $\mathcal{APP}$ and country-level indices of financial development. Section 6 extends the model to allow for trade costs, global value chains, and imperfect capital-market integration. Section 7 concludes.

[4]A related literature has studied the role of inventories and of 'just-in-time' techniques in shaping specialization and the reactions of firms to trade shocks. The work of Alessandria et al. (2011), and the more recent contributions by Ferrari (2022), Pisch (2020) and Carreras-Valle (2022) exemplify some of the insightful work in this area. Our model also features inventories, and the measure of the $\mathcal{APP}$ in Antràs and Tubdenov (2025) is indeed based on inventory data, but our framework abstracts from active inventory management decisions. We believe this would be a fruitful extension of our theoretical framework.

2 Closed-Economy Model

We begin our theoretical analysis by presenting the model in its closed-economy version. There are three main building blocks in our framework. First, we develop a partial-equilibrium 'Austrian' model of firm behavior with an explicit treatment of the duration of the production process, and its associated working-capital needs. Second, we embed this model in a general-equilibrium framework with a continuum of finished goods, along the lines of Dornbusch et al. (1977). Third, we close the model by incorporating intertemporal preferences and a resulting supply of financial capital.

Environment Time evolves continuously. The economy is populated by a constant mass L of finitely-lived individuals. All agents are endowed with one unit of labor services which they supply inelastically to the market. Consumers have preferences over a continuum of final goods produced with labor in a time-phased manner. Production technologies (to be described below) are freely available to all agents in the economy, and perfect competition prevails in all markets, including the labor and capital markets.

2.1 Time and Production: 'Austrian' Partial Equilibrium

Consider a production process taking place in continuous time over a potentially endogenous time interval $[0, T]$.[5] We will refer to T as the *length* of the production process. The mapping between inputs and output is governed by the production function

$$y(\boldsymbol{L}) = ZF(\boldsymbol{L}), \tag{1}$$

where $\boldsymbol{L}$ is the infinite-dimensional vector of inputs employed along the production process, i.e., $\boldsymbol{L} = \{\ell(t)\}_{t\in[0,T]}$.[6] The function $F(\boldsymbol{L})$ captures a production technology mapping labor $\boldsymbol{L}$ into output, and we assume it is homogeneous of degree one in the vector $\boldsymbol{L}$. The term Z is a productivity parameter. While it is initially treated as exogenous, we will later allow it to vary with the length of the production process T. Firms treat the wage rate w, the interest rate r, and the price of the good p as given. Anticipating our focus on stationary equilibria, we assume that these prices are time-invariant.

[5] We develop our model in discrete time in Online Appendix C.1, and explain there why we prefer our continuous-time formulation.

[6] It is straightforward to extend the framework to a situation in which production requires labor and materials. See Section 6.2 for an illustration of this in a variant of our model involving sequential production processes.

The total cost of production associated with this time-phased production process is given by

$$C\left(\boldsymbol{L}\right)=\int_{0}^{T}w\ell\left(t\right)e^{r\left(T-t\right)}dt,$$

where the last term in the integral captures the fact that wage payments in period t accumulate interest for a period $T-t$ before the firm collects revenue at the end of the process. Given a path of labor used along the production process, we define:

Definition 1. *The average period of production or $\mathcal{APP}$ is given by:*

$$\mathcal{APP}\left(\boldsymbol{L}\right)=\int_{0}^{T}\left(T-t\right)\frac{w\ell\left(t\right)e^{r\left(T-t\right)}}{C\left(\boldsymbol{L}\right)}dt. \tag{2}$$

The $\mathcal{APP}$ represents a weighted average temporal distance between the time $t\in[0,T]$ when labor expenditures are incurred and the time T marking the completion of the process and the sale of the good. The weights are determined by the shares of appropriately discounted labor expenditures at different points in time. This definition of the $\mathcal{APP}$ was first developed by Hicks (1939), but it is closely related to the one originally proposed by Böhm-Bawerk (1889), though in the latter's formulation the weights $\lambda\left(t\right)$ were simply assumed to equal the shares of employment at different points in time.[7]

The $\mathcal{APP}$ is determined by the length T of the production process but also by the path of labor input choices $\boldsymbol{L}$. For a given production length T, the average period of production is higher, the higher the share of labor expenditures that are incurred in earlier phases of production. The path of labor expenditures is in turn determined by a standard cost minimization problem

$$\begin{array}{ll}\min\limits_{\boldsymbol{L}} & \int_{0}^{T}w\ell\left(t\right)e^{r\left(T-t\right)}dt \\ \text{s.t.} & y\left(\boldsymbol{L}\right)\geqslant 1\end{array}. \tag{3}$$

The solution to this problem delivers a vector of unit labor requirements $\mathbf{a_L}\left(r\right)=\left\{a_L\left(t,r\right)\right\}_{t\in[0,T]}$, which are independent of the wage rate, but *do* depend on the interest rate. These unit labor requirements in turn determine the overall unit cost of production, and under zero profits, also

[7] To be more precise, Böhm-Bawerk (1889) defined the average period of production in discrete time as

$$\mathcal{APP}\left(\boldsymbol{L}\right)=\sum\nolimits_{t=0}^{T}\left(T-t\right)\frac{\ell\left(t\right)}{\sum\nolimits_{t=0}^{T}\ell\left(t\right)}.$$

The $\mathcal{APP}$ in equation (2) is also closely related to the concept of duration in finance (Macaulay, 1938). See Lewin and Cachanosky (2018) for a historical account of the concept of the average period of production.

the price of the good. We write this as

$$p(w,r) = c(w,r) = \int_0^T w a_L(t,r)\, e^{r(T-t)} dt. \tag{4}$$

Invoking the envelope theorem associated with the choice of $\ell(t)$ in (3), it is then straightforward to show (see Appendix A.1) that:

Proposition 1. *The unit cost function satisfies*

$$\frac{\partial \ln c(w,r)}{\partial r} = \mathcal{APP}.$$

In words, the average period of production (or $\mathcal{APP}$), as defined in equation (2), governs the percentage response of production costs to changes in the interest rate.[8] Intuitively, the higher is the $\mathcal{APP}$, the more will past labor expenditures accumulate interest, and thus the more will a higher interest rate increase production costs.

Endogenous Production Length So far we have taken the length of production T as exogenous, but it is straightforward to endogenize it. Böhm-Bawerk (1889) famously posited that more *roundabout* production processes tend to be associated with disproportionately higher labor efficiency. A simple way to capture the productivity benefits of longer production processes is to assume that productivity Z in equation (1) is increasing in T, or $Z = Z(T)$ with $Z'(T) > 0$.[9] Letting T be a choice variable in the cost-minimization problem (3) results in the following optimality condition for the choice of T:

$$T = \frac{\varepsilon_{Z,T} + \varepsilon_{F,T}}{\alpha_T + r}, \tag{5}$$

where $\varepsilon_{Z,T}$ is the elasticity of the function $Z(T)$ with respect to T (holding the vector $\boldsymbol{L}$ fixed), $\varepsilon_{F,T}$ is the elasticity of the function $F(\boldsymbol{L})$ with respect to T, and α_T is the ratio of labor expenditure at T relative to sale revenue, or $\alpha_T = w\ell(T) / (pZ(T)F(\boldsymbol{L}))$. Equation (5) hints at a negative relationship between T and r, which we will illustrate below with particular examples.[10] Although this endogenous determination of T affects the unit cost of production and the $\mathcal{APP}$, it is worth stressing that Proposition 1 holds *regardless* of whether T is exogenous or endogenous, a result that is straightforward to prove invoking the envelope theorem.

[8] Although this result is not particularly well known, it dates back (at least to) Weizsäcker (1971).

[9] We will later place further restrictions on $Z(T)$ to ensure that the second-order conditions for a maximum are satisfied in specific versions of the model.

[10] On the other hand, the optimal T is independent of the wage rate because working capital costs are proportional to labor expenditures, and as a result w/p and thus α_T are independent of w.

2.2 General Equilibrium Structure

We next embed our 'Austrian' model of firm behavior into a multi-sector general-equilibrium model, along the lines of the Ricardian model of Dornbusch et al. (1977). Consumers feature an instantaneous utility function over a continuum of sectors indexed by $v \in [0, 1]$. For simplicity, we follow Dornbusch et al. (1977) in assuming Cobb-Douglas preferences so that

$$\ln U = \int_0^1 \beta_v \ln c\,(v)\, dv. \tag{6}$$

Each sector v thus captures a share β_v of consumer spending at any point in time.[11] All goods are produced with labor according to 'Austrian' production technologies of the type described in the last section. Nevertheless, sectors vary in their particular production functions, or $y_v = Z_v\,(T_v)\, F_v\,(\boldsymbol{L}_v)$.

We restrict attention to a stationary equilibrium in which firms in each sector simultaneously carry out various production processes at various phases of completion. More specifically, at each instant t, firms (i) begin the production of y_v units of sector v goods, (ii) continue to add value to the goods begun in previous periods $t' \in (t - T_v, t)$, and (iii) complete the production of the y_v goods that they had begun at $t - T_v$. It would of course be more desirable to characterize the transitional dynamics of the model, rather than focusing on a comparison of steady-states. Yet, as in many models with heterogeneous capital goods, doing so is hard and beyond the scope of this paper.

Allocation of Labor Across Sectors Imposing goods-market clearing (i.e., $p_v y_v = \beta_v I$, where I is aggregate income), the zero-profit conditions (4), and labor-market clearing, we obtain the following stationary share of labor allocated to sector v

$$L_v = \frac{\Lambda_v\,(r)}{\int_0^1 \Lambda_v\,(r)\, dv} L, \tag{7}$$

where the term $\Lambda_v\,(r)$ shapes the share of labor going to sector v according to

$$\Lambda_v\,(r) = \beta_v \frac{\int_0^{T_v} a_{Lv}\,(t, r)\, dt}{\int_0^{T_v} a_{Lv}\,(t, r)\, e^{r(T_v - t)} dt}. \tag{8}$$

In this last expression, remember that $a_{Lv}\,(t, r)$ is the unit labor requirement associated with labor at instant $t \in [0, T_v]$ in sector v. For a given interest rate r, equations (7) and (8) indicate that relatively less labor tends to be allocated to sectors with longer production lengths T_v,

[11] Relaxing this assumption would greatly complicate characterizing the equilibrium, as sectoral spending would depend on goods – and thus – factor prices.

and especially so the higher is the interest rate r. Nevertheless, it is not possible to obtain general results regarding how the allocation of labor across sectors is shaped by the interest rate r. First, for certain specifications of the technology in equation (1), it is certainly possible for $\Lambda_v(r)$ to be increasing in r for a range of interest rates.[12] Second, the impact of r on $\Lambda_v(r)$ is further complicated whenever T_v is endogenous and negatively affected by interest rates. In Section 3, we will develop a particularly tractable special case in which $\Lambda(r)$ will turn out to be *independent* of r.

The zero-profit conditions (4) constitute a measure one of equations pinning down the wage rate w and the prices of all goods in terms of a numéraire. Choosing as the numéraire the ideal price index associated with the Cobb-Douglas preferences in (6), we obtain a real wage equal to

$$\ln w = \psi - \int_0^1 \beta_v \ln \left(\int_0^{T_v} a_{Lv}(t,r)\, e^{r(T_v - t)} dt \right) dv, \tag{9}$$

for some positive constant ψ. This expression solves for the real wage rate as a function of the interest rate r and additional terms that are independent of the wage rate. Invoking the envelope theorem associated with the choices of $a_{Lv}(t,r)$ and T_v, it is then straightforward to verify that:

Proposition 2. *The real wage w satisfies*

$$\frac{\partial \ln w}{\partial r} = - \int_0^1 \beta_v \mathcal{APP}_v dv < 0, \tag{10}$$

and thus there is a negative relationship between the wage rate w and the interest rate r.

Equation (10) not only demonstrates that the lower is the interest rate r, the higher is the wage rate w, but it also shows that the effect is larger, the larger are the average periods of production of the various sectors in the economy. Intuitively, when production processes take a long time to complete, more interest accumulates during production, and the lower is the resulting demand for labor.

Capital Demand We finally turn to discussing equilibrium in the capital market, beginning with the demand side of that market. Working-capital needs associated with a production process

[12]Consider a sector with fixed completion date $T_v > 0$ and (for simplicity) two dated labor inputs: "early" labor ℓ_{0v} used at $t = 0$ and "late" labor ℓ_{Tv} used at $t = T_v$. Let output be produced with a CES technology

$$y_v = \left(\ell_{0v}^{\frac{\varepsilon_v - 1}{\varepsilon_v}} + \ell_{Tv}^{\frac{\varepsilon_v - 1}{\varepsilon_v}} \right)^{\frac{\varepsilon_v}{\varepsilon_v - 1}}, \qquad \varepsilon_v > 1.$$

The resulting function $\Lambda_v(r)$ is *not* monotone: one can verify $\Lambda_v'(0) < 0$, while $\Lambda_v'(r) > 0$ for sufficiently large r (e.g., a sufficient condition is $rT_v > \ln\left((\varepsilon_v + 1) / (\varepsilon_v - 1)\right)$).

in sector v completed up to instant s are given by $\int_0^s w\ell_v(t)\, e^{r(s-t)}dt$, so total working-capital needs in sector v can be expressed (after a change in the order of integration) as

$$K_v^d = \int_0^{T_v} w\ell_v(t)\, \frac{e^{r(T_v-t)}-1}{r} dt. \tag{11}$$

The same expression could be derived when calculating the present discounted value of the revenue that would be obtained by selling all the goods in process at various degrees of completion. More specifically, we have $K_v^d = y_v \int_0^{T_v} p_v^k(s,r)\, ds$, where

$$p_v^k(s,r) = \int_0^s w a_{Lv}(t,r)\, e^{r(s-t)}dt \tag{12}$$

is the price of a capital good of age $s \leqslant T_v$. We can thus interpret sectoral capital demand K_v^d as the sum of the values of all goods in process in a given sector.

Because production requires labor and capital, we can also decompose the unit cost function in (4) as

$$c_v(w,r) = w a_{Lv}(r) + r a_{Kv}(w,r), \tag{13}$$

where the total unit labor requirement $a_{Lv}(r)$ in sector v is given by

$$a_{Lv}(r) = \int_0^{T_v} a_{Lv}(t,r)\, dt,$$

and where the total unit capital requirement in sector v is given by

$$a_{Kv}(w,r) = \int_0^{T_v} a_{Kv}(w,r,t)\, dt = \int_0^{T_v} w a_{Lv}(t,r)\, \frac{e^{r(T_v-t)}-1}{r} dt.$$

One can easily verify that this is indeed in accordance with equation (11) given that $a_{Kv}(w,r) = K_v^d / y_v$.

Although equation (13) seems to define a cost function in terms of two homogeneous factors, labor and capital, in our model sectoral capital demand *cannot* be calculated by differentiating the cost function with respect to the interest rate r. More specifically, for $r > 0$, we have that

$$\frac{\partial c_v(w,r)}{\partial r} = \mathcal{APP}_v \times c_v(w,r) = \int_0^{T_v} a_{Kv}(w,r,t)\, \frac{r(T_v-t)}{1-e^{-r(T_v-t)}} dt > a_{Kv}(w,r) = \frac{K_v^d}{y_v},$$

where the strict inequality follows from $x/(1-e^{-x}) > 1$ for all $x > 0$.

This 'failure' of Shephard's lemma suggests that interpreting working capital as a standard homogeneous neoclassical factor of production may be problematic in our setting. Indeed, as mentioned above, working-capital needs are tightly related to the value of goods in process, of which there are a continuum differing in their levels of completion. An increase in the interest rate increases the price of work-in-process goods, but it does so disproportionately for goods close to completion, which have cumulated more interest. This is why the unit cost $c(w,r)$ of completed products goes up by *more* than the overall increase in the value of all capital goods at various levels of completion. We will return to this point later in the paper, when it becomes essential to understanding certain results.

There are only two special situations in which Shephard's lemma correctly delivers the demand for financial capital. First, when $r \to 0$, capital remuneration vanishes, and the relative price of all capital goods is fixed (see equation (12)). As shown in Weizsäcker (1971), what is key in this special case is *not* that the interest rate is 0, but rather that it equals the growth rate of the economy g (which in our model is 0). As long as $r = g$ (i.e., under the 'golden rule'), the relative price of all capital goods will be independent of the interest rate.[13] A second case in which Shephard's lemma holds occurs whenever time is discrete and production takes exactly *one* period in all sectors. In that case, there is again a single type of homogeneous capital good in the economy (see Online Appendix C.1).

Beyond creating a wedge between capital intensity and the derivative of the cost function with respect to the interest rate, the above failure of Shephard's lemma also generates the possibility of the ranking of sectors in terms of capital intensity differing from the ranking of sectors in terms of their average period of production. In other words, for certain paths of labor use, it may be the case that a sector features a higher $\mathcal{APP}_v$ *yet* a lower ratio $a_{Kv}(w,r)/a_{Lv}(r)$ than another sector. We refer to this possibility as the existence of time-versus-capital intensity ranking reversals:

Definition 2. *A time-versus-capital intensity ranking reversal occurs whenever for two sectors v and v', $\mathcal{APP}_v < \mathcal{APP}_{v'}$ but $a_{Kv}(w,r)/a_{Lv}(r) > a_{Kv'}(w,r)/a_{Lv'}(r)$, for some w and r.*

This possibility is somewhat reminiscent of the possibility of factor intensity reversals in the Heckscher-Ohlin model, but it is worth stressing that time-versus-capital intensity ranking reversal can occur even in the absence of either capital intensity reversals or $\mathcal{APP}$ reversals (see Definition 3 below). We will illustrate this possibility with an example in Section 4.4.

Another peculiar aspect of our model is that it is not obvious that sectoral capital demand is decreasing in the interest rate r. Although lower interest rates tend to shift labor towards relatively upstream instants of production and they also tend to increase production lengths T

[13]More specifically, if wages grow at rate g, then the price of a capital good of age $s \leqslant T_v$ is given by

$$p_{kv}(s,r) = \int_0^s w_s a_{Lv}(t,r)\, e^{(r-g)(s-t)} dt,$$

and for $r = g$, the envelope theorem associated with the cost-minimization problem (3) indicates that $p_{kv}(s,r)$ is independent of the interest rate.

(when these are endogenous), cumulated interest will also tend to be lower with lower interest rates, so the balance of these effects is potentially ambiguous. Nevertheless, we will develop examples below in which K_v^d is unambiguously decreasing in the interest rate.

Aggregating K_v^d across sectors, we can finally compute aggregate capital demand in our closed economy as

$$K^d = \int_0^1 K_v^d dv.$$

As long as K_v^d is (weakly) decreasing in the interest rate r in all sectors v, the aggregate demand for capital K^d will also be decreasing in r.

We finally invoke the zero-profit condition for all goods to show that

$$GDP = \int_0^1 p_v y_v dv = wL + \int_0^1 \int_0^T \left(e^{r(T_v - t)} - 1\right) w\ell_v(t)\, dt dv = wL + rK^d,$$

which of course constitutes the standard national identity equating the value of final-good production to national income, the latter being the sum of labor and capital income.

2.3 Intertemporal Preferences and the Supply of Capital

So far, we have treated the interest rate r as exogenous, but we now close our model by introducing a supply for capital. We do so following the stylized overlapping generations model in Caballero et al. (2008), though similar results could be obtained in richer models of intertemporal choice.

Remember that we have assumed that the economy is populated by a constant mass L of finitely-lived individuals. We now further assume that this is so because agents are born at a rate ρ per unit of time and die at the same rate. Furthermore, we follow Caballero et al. (2008) in making the strong assumption that agents save all their income during their lifetime, and consume only when they (are about to) die, capturing the fact that agents save to provide for their retirement. For simplicity, we assume that the Cobb-Douglas aggregator of all goods in (6), which above we chose to be the numéraire, is the only store of value, so agents save in terms of units of (or claims on) this composite good and lend it to firms seeking to pay workers before selling their goods. If $K^s(t)$ denotes aggregate savings (or financial capital) cumulated up to date t, then aggregate consumption at time t is $\rho K^s(t)$. The parameter ρ is thus inversely related to the aggregate propensity to save of this economy, and thus can also be interpreted as reflecting the 'impatience' of agents.

Although the capital market is perfectly competitive, we introduce frictions in lending in the form of monitoring costs. More specifically, we assume that collecting interest demands that lenders incur monitoring costs m per unit of capital lent. We interpret m as an inverse measure of the *financial development* of the economy. In analogy to iceberg trade costs, we model these monitoring costs as 'melted' resources, as in Ramos-Menchelli and Van Doornik (2022). We assume that this monitoring technology is widely available in the population, so whether these lenders are individuals, firms, or banks is immaterial. We discuss the implications of relaxing this assumption briefly in Section 6.3 and more extensively in a previous version of the paper (Antràs, 2023a).

For an exogenously given initial capital stock K_0, aggregate capital evolves according to

$$\frac{dK^s(t)}{dt} = w(t)L + (r(t) - m)K^s(t) - \rho K^s(t).$$

Therefore, in a steady state in which all variables are time invariant, we must have

$$K^s = \frac{wL}{\rho + m - r}. \tag{14}$$

The supply of capital is thus proportional to labor income, with the factor of proportionality being positively affected by the interest rate r and negatively affected by the parameter ρ governing the impatience of agents and by monitoring costs m, which depress the return to capital for savers.[14]

With this capital supply at hand, it is straightforward to characterize the equilibrium interest rate à la Metzler (1951). Notice, in particular, that combining (11) and (14), we have

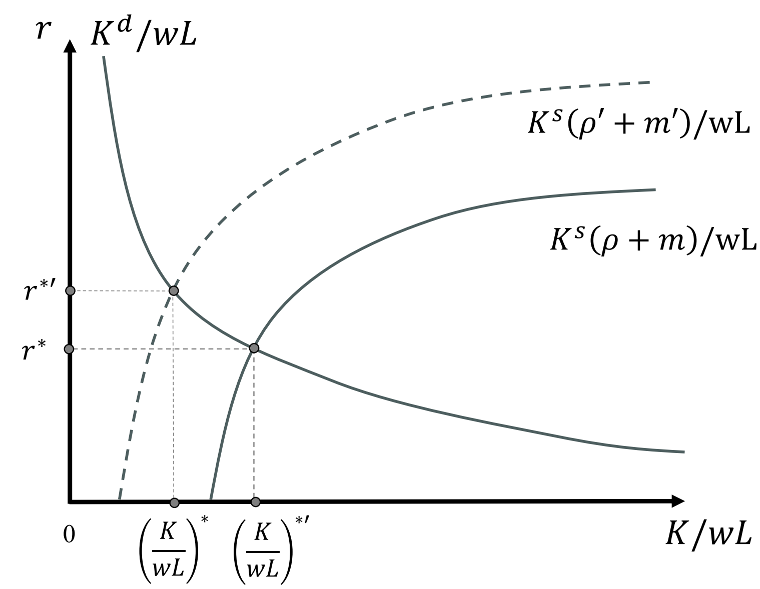
$$\frac{K^d}{wL} = \int_0^1 \int_0^{T_v} \frac{\ell_v(t)}{L} \frac{e^{r(T_v - t)} - 1}{r} dt dv = \frac{1}{\rho + m - r} = \frac{K^s}{wL}. \tag{15}$$

The ratio $K^s/wL = 1/(\rho + m - r)$ is monotonically increasing in r, starting at $1/(\rho + m)$ when $r = 0$, and with an asymptote to infinity when $r \to \rho + m$. If the aggregate capital demand is downward sloping and is sufficiently large when $r \to 0$, the equilibrium will exist and will be unique. This is the case depicted in Figure 1, which also illustrates the effects of an increase in impatience ρ or in monitoring costs m on that equilibrium. Even when the aggregate capital demand is not downward sloping, as long as it crosses the aggregate supply schedule only once from above, there will exist a unique and stable equilibrium.

The framework in Caballero et al. (2008) is special in many ways, but the fact that steady-state aggregate savings are proportional to labor income and can be expressed as $K^s = wL \times \sigma(r, \rho, m)$ – for some function $\sigma(r, \rho, m)$ of the interest rate and of preference or financial frictions parameters – is a feature of many OLG models. In our framework, we have $\sigma(r, \rho, m) = 1/(\rho + m - r)$, but our key results are independent of the precise function

[14] Note that ρ reflects the degree of impatience of agents, but it does *not* correspond to a discount rate, so the fact that, when m is small, we must have $r < \rho$ does not necessarily imply a form of dynamic inefficiency.

Figure 1: Equilibrium in the Capital Market



Notes: The figure plots the demand and supply for capital. An increase in impatience ρ or in monitoring costs m shifts capital supply to the left and leads to a higher equilibrium interest rate.

$\sigma(r, \rho, m)$.

The fact that capital supply is endogenous in our framework and responds to both wages and interest rates highlights another key difference between our model and the standard Heckscher-Ohlin model, in which the supply of capital is typically assumed to be fixed, even when the model is often interpreted to be a model of the 'long run'. The elastic capital supply in our framework will somewhat complicate the analysis of the free trade equilibrium, and also its comparison with the Heckscher-Ohlin model. For this reason, at times we will also characterize the equilibrium of our model for the case in which the supply of capital is *fixed* and given by K^s. In Online Appendix C.3, we develop a stylized OLG framework that delivers such an interest-rate and income-inelastic aggregate savings function as an equilibrium outcome.

3 An Illustrative Example: Wicksell's Timber Model

With the goal of providing an illustration of the workings of our model, in this section we consider a highly tractable special case – the so-called point-input point-output framework – akin to that first developed by Wicksell (1934), and further explored by Metzler (1950), Swan (1956), Cass (1973) and Findlay (1978), among others. More specifically, we assume that the production technology $F_v(\boldsymbol{L}_v)$ is such that inputs are only needed at the very beginning of production, i.e., $t = 0$, and thus an interval T_v before completion.[15] According to the definition of the average period of production in equation (2), this immediately delivers $\mathcal{APP}_v = T_v$. The $\mathcal{APP}$ thus coincides with the interval of time over which production takes place. This extreme version of the 'Austrian' model of production is typically coupled with the assumption that the technology function $Z_v(T_v)$ is increasing in T_v, thereby capturing the benefits of 'maturation.' We further assume $Z_v''(T_v)/Z_v'(T_v) < Z_v'(T_v)/Z_v(T_v)$ to ensure that the second-order conditions for profit-maximization are met.

This variant of our model maps particularly closely to the production of timber, in which trees are planted by labor at some initial instant, and one needs only wait for trees to grow, with no further labor input needed (this literature often ignores the labor needed to cut down the tree, but see our treatment in Section 4.4). With that in mind, we will often refer to this model as *Wicksell's timber model*. Nevertheless, this stylized model is also a reasonable approximation to the production of wine or distilled spirits, which also benefit from a process of maturation.[16]

Given our constant returns to scale assumption, we must necessarily have $F_v(\boldsymbol{L}_v) = \kappa \ell_v(0)$, for some κ independent of the choice of $\ell_v(0)$, which we set to 1 without loss of generality. The choice of T_v then solves:

$$\max_{T}\ \pi = p_v Z_v(T_v)\,\ell_v(0)\,e^{-rT_v} - w\ell_v(0).$$

Regardless of the choice of $\ell_v(0)$, the first-order condition for the choice of T_v is given by

$$\frac{Z_v'(T_v)}{Z_v(T_v)} = r. \tag{16}$$

Equation (16) is a well-known formula in Austrian models of capital, though it was first derived by Jevons (1871, p. 245). It indicates that, at the optimal production length, the growth of labor productivity is equated to the interest rate. This condition is naturally a special case of the more general solution (5) because, in this Wicksellian economy, the function $F_v(\boldsymbol{L}_v)$ does not depend on T directly ($\varepsilon_{F,T} = 0$) and the share of labor expenditure at T is 0 ($\alpha_T = 0$). Given the second order condition $Z_v''(T_v)/Z_v'(T_v) < Z_v'(T_v)/Z_v(T_v)$, it immediately follows that the average period of production ($\mathcal{APP}_v$) is decreasing in the interest rate r.

The Constant-Elasticity Case: the Wicksell-Swan Model A particularly useful formulation of the schedule $Z_v(T_v)$ is a constant-elasticity function $Z_v(T_v) = (T_v)^{\zeta_v}$ for a vector of constants $\boldsymbol{\zeta} = \{\zeta_1, \zeta_2, ..., \zeta_v\}$ with $\zeta_v \geqslant 0$ for all v. These constants ζ_v govern the

[15] In terms of the more general specification in the last section, this point-input point-output formulation boils down to assuming that $\ell_v(t) = L_v\delta(t)$, where $\delta(t)$ is the Dirac delta function.

[16] Indeed, Antràs and Tubdenov (2025) find that distilleries (NAICS code 312140) and wineries (NAICS code 32130) are the two sectors featuring the longest $\mathcal{APP}$ in US manufacturing.

importance of 'maturation' for productivity, or the *time intensity* of production for the various goods v. We further assume that the sectors can be re-ordered such that ζ_v is a continuously differentiable decreasing function of v taking bounded values. This constant-elasticity functional form was also invoked in the Appendix of Swan (1956) (more on this below), so we will refer to this special case of the model as the *Wicksell-Swan model*.[17] With this functional form, the second-order condition is always met for $\zeta_v \geqslant 0$, and equation (16) delivers

$$T_v^* = \zeta_v / r.$$

The optimal $\mathcal{APP}_v$ is thus increasing in the time intensity of good v, and is decreasing in the interest rate r. A convenient feature of this constant-elasticity case is that the resulting cumulative interest $e^{rT_v^*}$ is independent of the interest rate r (and equal to e^{ζ_v}), which allows for a particularly transparent derivation of the general equilibrium of the model. More specifically, the steady-state allocation of labor across sectors in equations (7) and (8) reduces to

$$L_v = \frac{\beta_v e^{-\zeta_v}}{\int_0^1 \beta_v e^{-\zeta_v} dv} L, \tag{17}$$

and is thus independent of the interest rate. The wage rate in (9) is in turn given by

$$\ln w = \int_0^1 \beta_v \left(\zeta_v \ln \left(\zeta_v / r \right) - \zeta_v - \ln \beta_v \right) dv,$$

from which Proposition 2 can be readily verified. Finally, sectoral capital demand takes the simple form

$$K_v^d = \frac{w L_v}{r} \frac{1 - e^{-\zeta_v}}{e^{-\zeta_v}}, \tag{18}$$

and is monotonically decreasing in the interest rate (since L_v in (17) is independent of r). Sectoral capital demand is also monotonically increasing in the time intensity of that sector (as captured by ζ_v), which implies that time-versus-capital intensity ranking reversals can *never* occur under this specification. Naturally, aggregate capital demand $K^d = \int_0^1 K_v^d dv$ is also decreasing in the interest rate in that case. Finally, we can invoke equation (15) to solve for the equilibrium interest rate in closed form as

$$r = (\rho + m) \int_0^1 \beta_v \left(1 - e^{-\zeta_v} \right) dv. \tag{19}$$

It is easy to generalize the above constant-elasticity, point-input, point-output model to

[17] We are grateful to Barbara Spencer (Trevor Swan's daughter) for making an electronic version of Swan (1956) available to us.

an environment in which labor is used at a uniform level along the whole production process. We develop this case in Online Appendix C.2. There, we show that the optimal production length is given by $T_v^* = \tilde{\zeta}_v / r$, where $\tilde{\zeta}_v$ is a monotonically increasing transformation of ζ_v. The remaining equilibrium conditions are also analogous to those of Wicksell's timber model, with the allocation of labor across sectors independent of the interest rate, and with a constant share of income accruing to labor and capital in all sectors (as in equation (18)).

Timeless Representation Equation (18) suggests that, in this constant-elasticity case, the sectoral capital and labor shares are constant and independent of wages and interest rates. This raises the possibility of this version of the model being isomorphic to a *timeless* neoclassical model in which technologies are Cobb-Douglas in labor and some aggregate of physical capital which earns a return r in all sectors. More broadly, this result may suggest the possibility to reduce our model with more general time-phased production structures in Section 2 to a *timeless* neoclassical model with two homogeneous factors of production, as in the Heckscher-Ohlin model. In his quest to microfound an aggregate Cobb-Douglas production function, Swan (1956) studied this possibility for the case of a one-sector model, and concluded that it is indeed possible to specify output as a Cobb-Douglas aggregator of labor and a quantity aggregate of capital (or *physical* capital). Nevertheless, Swan (1956) also showed that the return to this homogeneous physical capital is *not* equal to the interest rate r. Intuitively, changes in the interest rate not only shape the optimal marginal product of physical capital but also lead to (re)valuation effects of the inventory of capital goods, a so-called *Wicksell effect* (see also Metzler, 1950).

For the case of our multi-sectoral Wicksellian economy (with labor used only at the beginning of production), we can show that when $Z_v(T_v) = (T_v)^{\zeta_v}$, the general equilibrium of our model is indeed isomorphic to that of a *timeless* multi-sectoral neoclassical model in which technology in sector v is given by

$$y_v = \left(\tilde{K}_v\right)^{1-e^{-\zeta_v}} (L_v)^{e^{-\zeta_v}}, \tag{20}$$

in which labor L_v commands a wage w in all sectors, and in which $\tilde{K}_v$ is a sectoral stock of *physical* capital. This stock can be computed by deflating the stock of financial capital K_v in sector v by a *sector-specific* price of a capital $\tilde{p}_v^k$ given by

$$\tilde{p}_v^k = \vartheta_v w r^{g(\zeta_v)}, \tag{21}$$

where $\vartheta_v > 0$ is a positive constant and $g(\zeta_v)$ is a positive and increasing function.[18] Financial capital obtains a return r in all sectors, but physical capital is instead remunerated at a rate of

[18]More specifically

$$\vartheta_v = (\zeta_v)^{-\frac{\zeta_v}{1-e^{-\zeta_v}}} \left(e^{\zeta_v} - 1\right)$$

$r\tilde{p}_v^k$ per unit in sector v. Under this timeless formulation, Shephard's lemma *does* apply, and the sectoral demands for 'physical' capital $\tilde{K}_v$ and labor L_v can be obtained by differentiating the Cobb-Douglas cost function dual to (20) with respect to $r\tilde{p}_v^k$ and w, respectively.[19]

These results highlight that our model has a 'specific factors' or Ricardo-Viner flavor, in the sense that it exhibits differences across sectors in the return to physical capital. Nevertheless, unlike in a Ricardo-Viner model, the sectoral capital stocks are *not* exogenously given; instead they are determined as part of the equilibrium. Indeed, the interest rate continues to be determined by the equality of the aggregate demand $K^d = \int_0^1 \tilde{p}_v^k \tilde{K}_v dv$ and aggregate supply K^s of *financial* capital, resulting in equation (19). This equilibrium interest rate then determines the wage rate w and the relative price of capital $\tilde{p}_v^k$ in sector v from equation (21). Dividing the allocation of financial capital K_v^d in (18) by this price of capital, results in the equilibrium allocation of physical units of capital $\tilde{K}_v$ in each sector.

The bottom line is that although our model features free mobility of labor and of financial capital across sectors, the heterogeneous nature of goods in process leads to a model that behaves quite differently from a model with free mobility of labor and (a homogeneous type of) *physical* capital. This suggests that the determinants and consequences of international specialization might be different in our model relative to the Heckscher-Ohlin model. We next turn to studying these issues formally.

4 Free-Trade Equilibrium

Consider a world economy in which two economies – Home and Foreign – of the type described above, engage in international trade. We return to our more general framework in Section 2, but we make a number of simplifying assumptions to streamline the discussion of the model's key results. First, we assume that countries differ in the 'impatience' of their agents, their monitoring costs and (possibly) their population sizes. In particular, we assume that agents at Home are sufficiently more patient than those in Foreign or that financial development is sufficiently higher at Home than in Foreign to ensure that $\rho^H + m^H < \rho^F + m^F$. To keep the notation tidy, we introduce the notation $\phi^j \equiv \rho^j + m^j$, so our running assumption is $\phi^H < \phi^F$.

Second, we assume that all production technologies and all schedules $Z_v(T_v)$ are common

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and

$$g(\zeta_v) = \frac{e^{-\zeta_v} + \zeta_v - 1}{1 - e^{-\zeta_v}}.$$

[19] For the version of our model with a uniform use of labor along the production process, we can show a completely analogous result, though the exponents on capital and labor in equation (20) are replaced with $1 - \tilde{\zeta}_v / \left(e^{\tilde{\zeta}_v} - 1\right)$ and $\tilde{\zeta}_v / \left(e^{\tilde{\zeta}_v} - 1\right)$, respectively, and the sectoral price of capital is of course given by a different formula (see Online Appendix C.2).

in both countries, again in analogy to the assumption of identical technologies in the Heckscher-Ohlin model. In light of equation (15), and because countries share the same technologies, Home necessarily features a lower interest rate ($r^H < r^F$) and a higher wage rate ($w^H > w^F$) under autarky, regardless of the labor endowments L^H and L^F of the two countries. The reason for this is that both capital demand *and* capital supply are proportional to population. At times, we will consider the possibility that capital supply is instead fixed and independent of wages and interest rates, in analogy to the treatment of factor endowments in the Heckscher-Ohlin model. In such a case, we will simply assume that $K^H/L^H > K^F/L^F$ because this is the condition that will ensure (in 'well-behaved' economies) a lower interest rate and a higher wage rate at Home relative to Foreign *under autarky* (see Online Appendix C.3).

Third, we assume that goods can only be traded once they are completed, so there is no international trade in semi-finished products. Conversely, international trade in finished consumer goods incurs no trade costs. In Section 6, we will develop extensions with free trade in finished *and* semi-finished goods, and also with costly trade in both types of goods.

Finally, we rule out the possibility of international borrowing or lending by assuming that if domestic lenders tried to collect interest from foreign borrowers, they would need to incur prohibitively high monitoring costs. We will briefly relax this assumption in Section 6.

Having laid out our assumptions, we next turn to characterizing the open-economy equilibrium. We will initially impose that factor prices differ across countries even in a free trade equilibrium, but we will later consider the possibility of trade bringing about factor price equalization (FPE, hereafter).

4.1 Pattern of Comparative Advantage

Consider a free trade equilibrium in which Home and Foreign differ in their factor prices w^j and r^j. Given equation (4), the minimum price at which country j can produce good v can be expressed as a linear function of the wage rate

$$p_v^j = \tilde{a}_{Lv}^j \left(r^j\right) w^j, \tag{22}$$

where $\tilde{a}_{Lv}^j (r^j)$ is given by $\tilde{a}_{Lv}^j (r^j) = \int_0^T a_{Lv}^j (r^j, t)\, e^{r(T-t)} dt$. In light of Proposition 1, $\tilde{a}_{Lv}^j (r^j)$ satisfies

$$\frac{\partial \ln \tilde{a}_{Lv}^j (r^j)}{\partial r^j} = \mathcal{APP}_v^j \,. \tag{23}$$

In words, equations (22) and (23) indicate that our model reduces to a Ricardian model of trade in which comparative advantage is governed by effective labor productivity differences across countries, and in which these effective labor productivity differences are shaped by the interaction of interest rate differentials across countries and $\mathcal{APP}$ differences across sectors.

To be more formal, let us define the concept of an 'absence of $\mathcal{APP}$ reversals':

Definition 3. *Reversals in the average period of production do* not *occur if for any two sectors v and v' either (i) $\mathcal{APP}_v^j < \mathcal{APP}_{v'}^j$ for all r^j or (ii) $\mathcal{APP}_v^j > \mathcal{APP}_{v'}^j$ for all r^j.*

With this definition at hand, we can state our main result characterizing comparative advantage in our model:

Proposition 3. *As long as there are no $\mathcal{APP}$ reversals, the country with the lower interest rate necessarily has comparative advantage in sectors with relatively long average production periods ($\mathcal{APP}$s).*

The key for this result is that we can express the ratio of Foreign and Home prices for any two goods v and v' as

$$\frac{p_v^F/p_v^H}{p_{v'}^F/p_{v'}^H} = \frac{\tilde{a}_{Lv}^F\left(r^F\right)/\tilde{a}_{Lv}^H\left(r^H\right)}{\tilde{a}_{Lv'}^F\left(r^F\right)/\tilde{a}_{Lv'}^H\left(r^H\right)}, \tag{24}$$

and equation (23) indicates that $\tilde{a}_{Lv}^j\left(r^j\right)$ is log-supermodular in r^j and $\mathcal{APP}_v^j$. Coupled with the lack of $\mathcal{APP}$ reversals, this ensures that the high interest rate country features disproportionately higher prices in sectors with longer $\mathcal{APP}$s (cf., Costinot, 2009a). This result is particularly straightforward to illustrate in Wicksell's timber model, for which equation (24) can be expressed as

$$\frac{p_v^F/p_v^H}{p_{v'}^F/p_{v'}^H} = e^{\left(r^F-r^H\right)\left(T_v-T_{v'}\right)},$$

and thus, if $r^H < r^F$, the relative cost advantage of Home is larger in high-T (i.e., long-$\mathcal{APP}$) sectors. It is also worth emphasizing that because Proposition 1 holds even when endogenizing the production length T_v, Proposition 3 is also robust to endogenizing the choice of T_v. This can easily be illustrated for the special Wicksell-Swan model variant of our model with a constant-elasticity productivity function, for which equation (24) reduces to

$$\frac{p_v^F/p_v^H}{p_{v'}^F/p_{v'}^H} = \left(\frac{r^H}{r^F}\right)^{\zeta_v-\zeta_{v'}},$$

and thus the relative price at Home is indeed lower in more 'time intensive' sectors featuring higher values of ζ_v.

4.2 General Equilibrium

As long as wages and interest rates differ across countries and the average period of production is distinct in all sectors and satisfies the no $\mathcal{APP}$ reversals property, the ratio of relative price p_v^F/p_v^H can only be equal to 1 for at most one good v^*. To study this more formally, let us assume – without loss of generality – that the index v associated with goods is such that $\mathcal{APP}_v$ is *decreasing* in v. Next, as in Dornbusch et al. (1977), define the relative labor efficiency schedule

$$A(v) \equiv \frac{\tilde{a}^F_{Lv}\left(r^F\right)}{\tilde{a}^H_{Lv}\left(r^H\right)}. \tag{25}$$

As long as factor prices vary across countries, with $r^H < r^F$, the function $A(v)$ is necessarily decreasing in v, and there exists a unique $v^* \in (0,1)$, such that $A(v) > w^H/w^F$ for $v < v^*$ and $A(v) < w^H/w^F$ for $v > v^*$. In words, the low interest rate Home has a relative cost advantage in goods v with $v < v^*$, while Foreign has a relative cost advantage in all stages with $v > v^*$.

Next, consider labor demand and labor-market clearing. Following analogous derivations as those in the closed economy, we find that if Home produces goods $v \in [0, v^*]$ and Foreign produces goods $[v^*, 1]$, then:

$$\frac{w^H}{w^F} = \frac{\int_0^{v^*} \Lambda^H_v\left(r^H\right) dv}{\int_{v^*}^1 \Lambda^F_v\left(r^F\right) dv} \frac{L^F}{L^H} \equiv B\left(v^*\right), \tag{26}$$

where $\Lambda^j_v\left(r^j\right)$ is defined in equation (8), and (importantly) does not depend on wages.[20] The schedule $B(v^*)$ is an increasing function of the threshold stage v^* because as more stages are performed at Home, the numerator in (26) increases while the denominator decreases. Furthermore, when $v^* \to 0$, $B(v^*) \to 0$, and when $v^* \to 1$, $B(v^*) \to \infty$. For given interest rates $r^H < r^F$, equations (25) and (26) determine the unique equilibrium threshold v^* and relative wage w^H/w^F in a manner completely analogous to the analysis in Dornbusch et al. (1977). Figure 2 depicts such an equilibrium.

A key distinction between our framework and Dornbusch et al. (1977) is that labor efficiency, which determines the schedule $A(v)$, is *not* exogenous in our model: it is instead shaped by the average period of production in the various sectors, which is in turn shaped by the interest rates faced by producers, as made explicit in equation (25). Furthermore, the allocation of labor across sectors, which shapes the schedule $B(v^*)$, is also generally shaped by interest rates, as discussed in section 2.2. To complete the description of the equilibrium, we thus need to determine the interest rates in the two countries.

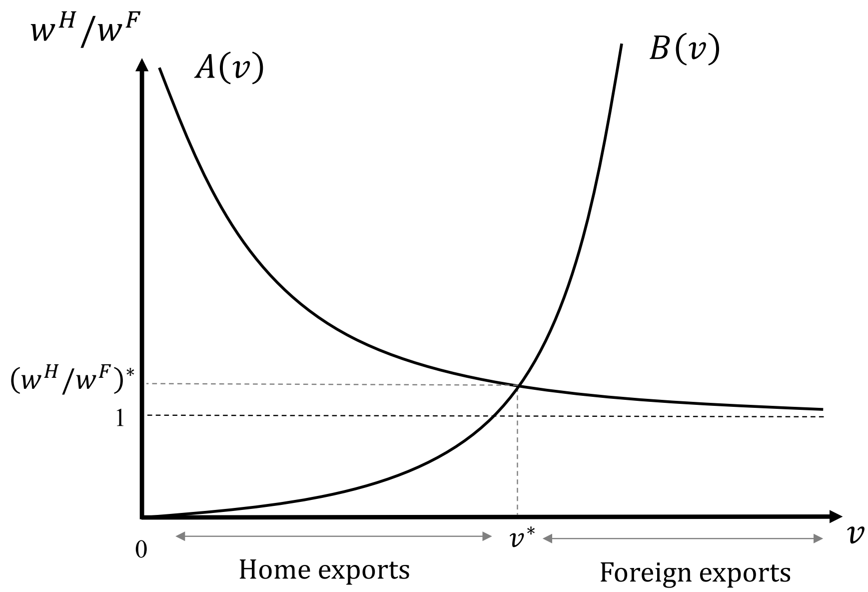
These equilibrium interest rates can be obtained by deriving capital demand and invoking capital-market clearing, as in the Heckscher-Ohlin model with a continuum of goods in Dornbusch

[20] More specifically, the use of labor by country j in sector v is $L^j_v = a^j_{Lv} y^j_v$, while revenue in that sector is $p^j_v y^j_v = \beta_v \left(w^H L^H + r^H K^H + w^F L^F + r^F K^F\right)$. Using the pricing equation (22), we then have

$$L^j_v = \frac{\Lambda^j_v}{w^j}\left(w^H L^H + r^H K^H + w^F L^F + r^F K^F\right).$$

Integrating over the set of goods that country j produces, and re-arranging, then delivers equation (26).

Figure 2: Trade Equilibrium with Factor Price Differences



Notes: The figure plots the trade equilibrium with no factor price equalization. Goods are ordered such that $\mathcal{APP}_v$ is decreasing in v. The schedules $A(v)$ and $B(v)$ are defined in equations (25) and (26), respectively.

et al. (1980). Following the same derivations as in the closed-economy version of our model, funding labor costs associated with producing good v in country j requires an amount of capital equal to

$$K_v^j = \int_0^{T_v} w^j \ell_v^j(t) \frac{e^{r^j(T_v - t)} - 1}{r^j} dt, \tag{27}$$

while capital supply continues to be given by equation (14). The Home and Foreign interest rates are then pinned down by

$$\int_0^{v^*} \frac{L_v^H}{L^H} \times \frac{K_v^H}{L_v^H} dv = \frac{K^{sH}}{L^H} = \frac{w^H}{\phi^H - r^H} \tag{28}$$

and

$$\int_{v^*}^1 \frac{L_v^F}{L^F} \times \frac{K_v^F}{L_v^F} dv = \frac{K^{sF}}{L^F} = \frac{w^F}{\phi^F - r^F}, \tag{29}$$

respectively. In these expressions, K_v^j (for $j = H, F$) is given in (27), and L_v^j can be computed analogously to equations (7) and (8) in our derivations of the closed-economy equilibrium. Equations (28) and (29) can be reduced to a system of two equations in two unknowns, r^H and r^F. This is because v^*, w^H and w^F can all be expressed as a function of r^H and r^F.[21]

As long as these equations deliver a lower interest rate at Home than in Foreign, the

[21] We have shown above that r^H and r^F uniquely pinned down v^* and w^H/w^F. In addition, our choice of

pattern of specialization will indeed be as described above, with Home fully specializing in the most time intensive goods (with $v < v^*$). Naturally, the fact that Home is more patient or more financially developed than Foreign (i.e., $\phi^H < \phi^F$) tends to make this equilibrium more likely, as is clear from the denominator of the right-hand-side of equations (28) and (29). Nevertheless, because Home specializes in long-$\mathcal{APP}$ sectors, and these sectors *tend to* feature higher capital intensities, the integrals in the left-hand-side of the above expressions – which are employment-weighted average capital intensities – will *tend to* be higher for Home than for Foreign, which constitutes a force towards a higher interest rate at Home.[22] Could these forces bring about factor price equalization? And could the equilibrium interest rate actually end up being higher in the more patient Home? We next seek answers to these questions.

4.3 Equilibrium in *Regular* Economies

We first characterize the nature of equilibria in 'well-behaved' or *regular* economies. By this we mean that we focus on specifications of technology such that: (i) capital intensity K_v^j/L_v^j is decreasing in the interest rate r^j; (ii) there are no $\mathcal{APP}$ reversals (see Definition 3), and (iii) there are no time-versus-capital intensity ranking reversals (see Definition 2). Our main result regarding the nature of the global general equilibrium in regular economies can be stated as follows (see Appendix A.4 for a formal proof):

Proposition 4. *In the absence of $\mathcal{APP}$ reversals and of time-versus-capital intensity ranking reversals, and provided that capital intensity K_v^j/L_v^j is decreasing in the interest rate, a world equilibrium is one of two types:*

a) *A complete-specialization equilibrium in which the country with the higher aggregate capital-labor ratio (K^s/L) features a lower interest rate and completely specializes in sectors with relatively long average production periods ($\mathcal{APP}$s); or*

b) *An equilibrium featuring factor price equalization and an indeterminate pattern of specialization.*

Furthermore, when aggregate capital-labor ratios are sufficiently similar in the two countries, the equilibrium is necessarily one with factor price equalization.

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numéraire implies

$$\int_0^{v^*} \beta_v \ln\left(w^H \tilde{a}_{Lv}^H\left(r^H\right)\right) dv + \int_{v^*}^1 \beta_v \ln\left(w^F \tilde{a}_{Lv}^H\left(r^F\right)\right) dv = 0,$$

which allows to solve for the level of both w^H and w^F as a function of v^*, w^H/w^F, r^H and r^F.

[22] We include the caveat 'tend to' because longer $\mathcal{APP}$s may not always be associated with higher capital intensities when there are time-versus-capital intensity ranking reversals (more on this below).

This result is analogous to the Heckscher-Ohlin theorem in neoclassical trade theory (see, in particular, Dornbusch et al., 1980), with variation in $\mathcal{APP}$s playing the role of variation in capital intensity in that neoclassical model. Indeed, the assumed absence of time-versus-capital intensity ranking reversals ensures that the relatively capital abundant country completely specializes in the set of relatively capital intensive sectors provided that factor price differences persist with free trade. Although the mechanics of the two models are similar, their empirical implications are distinct, as we will emphasize in Section 5.

Whenever differences in autarky interest rates across countries are small relative to the variation in capital and time intensity across sectors, equations (28) and (29) would appear to lead to a *higher* interest rate in the relatively capital abundant country producing time and capital intensive goods, thus contradicting the pattern of comparative advantage implicit in those equations. As stated in Proposition 4, the equilibrium in such situations is *not* one in which the relatively capital abundant country features a higher interest rate, but rather one in which both wages and interest rates are equalized across countries. Furthermore, as we show in Appendix A.4, these equalized factor prices correspond to those that would apply in the equilibrium of a fictional *integrated world economy* in which capital and labor are freely mobile across countries. The aggregate endowments of labor and capital of this fictional integrated economy are given by $\bar{L} = L^H + L^F$, and $\bar{K} = \bar{w}L^H / \left(\phi^H - \bar{r}\right) + \bar{w}L^F / \left(\phi^F - \bar{r}\right)$, where $\bar{w}$ and $\bar{r}$ are the equilibrium factor prices in the integrated equilibrium (cf., Dixit and Norman, 1980, or Helpman and Krugman, 1985).

Apart from some subtleties stemming from the fact that capital supply is endogenous in our framework, the equilibrium within the factor-price equalization (FPE) set is qualitatively identical to that in the Heckscher-Ohlin model with a continuum of goods in Dornbusch et al. (1980).[23] As is well known, the pattern of production and commodity trade in such a case is indeterminate, although full employment of both factors requires that, on balance, the more patient Home produces disproportionately relatively capital-intensive goods. More precisely, regardless of the specific pattern of specialization, the relatively capital-abundant country is necessarily a net exporter of capital services embodied in the traded goods, while the relatively capital-scarce country is a net exporter of labor services embodied in the traded goods.

The Role of the Assumptions It is important to highlight *what* exactly is ruled out by Proposition 4, and why the stated assumptions are necessary for the result. In a nutshell, the proposition rules out equilibria with factor price differences across countries in which the

[23]The endogeneity of capital supply leads us to state Proposition 4 in terms of *equilibrium* relative capital abundance, rather than in terms of the primitive differences in patience or financial development. Our empirical tests indeed employ equilibrium measures of financial capital abundance. In Online Appendix C.3, we discuss the equilibrium of our model whenever Home and Foreign capital supplies are fixed, as in the Heckscher-Ohlin model.

relatively capital scarce country specializes in sectors with long $\mathcal{APP}$s. As we have established in Proposition 3, assuming the absence of $\mathcal{APP}$ reversals is necessary to generate a tight link between differences in interest rates across countries and the pattern of international trade. We still need to ensure, however, that the country with the lower interest rate is indeed the one featuring a higher equilibrium aggregate capital labor ratio. This may seem a natural outcome, but as we will illustrate in the next section, one can construct equilibria that violate this property in the presence of time-versus-capital intensity ranking reversals; hence the need to assume the absence of these reversals in Proposition 3. We finally also need to assume that capital intensity is decreasing in the interest rate to again avoid equilibria in which the relatively capital scarce country may end up with a lower interest rate and comparative advantage in long-$\mathcal{APP}$ sectors by operating them at a disproportionately low capital intensity.

It is worth emphasizing that Proposition 4 discusses the nature of the potential type of equilibria, but does *not* establish the existence or uniqueness of these equilibria for given primitive parameter values. Doing so for general productivity functions $Z_v\left(T_v\right)$ and $F_v\left(\boldsymbol{L}_v\right)$ is complex because the system of equations (25)-(29) pinning down the equilibrium values of w^H/w^F, v^*, r^H, and r^F is hard to characterize generally. Having said this, in our numerical analysis of the model (see Section 4.4), we have verified the existence of a unique equilibrium in virtually all cases satisfying the assumptions in Proposition 4 are met.

An Illustration: Wicksell-Swan Model A particularly sharp characterization of the global equilibrium can be obtained for the special Wicksell-Swan model developed in Section 3. In that special case, the $\mathcal{APP}$ in sector v is simply given by $T_v^j = \zeta_v/r^j$ and thus there *cannot* be $\mathcal{APP}$ reversals.[24] Furthermore, capital intensity is given by $K_v^j/L_v^j = \left(w^j/r^j\right)\left(1-e^{-\zeta_v}\right)/e^{-\zeta_v}$, which is decreasing in the interest rate and in v. Hence, this variant of the model does not allow for capital intensity or time-versus-capital intensity ranking reversals, and all the conditions for Proposition 4 are thus satisfied.

Furthermore, the $A\left(v\right)$ schedule in such a case is a simple power function of the ratio of interest rates – $A\left(v\right) = \left(r^F/r^H\right)^{\zeta_v}$ – where remember that ζ_v is decreasing in v. The $B\left(v^*\right)$ schedule capturing goods- and labor-market clearing, turns out to be independent of interest rates and given by

$$B\left(v^*\right) = \frac{\int_0^{v^*}\beta_v e^{-\zeta_v}dv}{\int_{v^*}^1\beta_v e^{-\zeta_v}dv}\frac{L^F}{L^H}. \tag{30}$$

[24]Note that the optimal production length T_v^j is identical to the one under autarky. This is a more general feature of the equilibrium, as explained in Section 6.1.

Finally, the equations pinning down the interest rates r^H and r^F are given by

$$r^H = \phi^H \times \frac{\int_0^{v*} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{v*} \beta_v dv} \tag{31}$$

and

$$r^F = \phi^F \times \frac{\int_{v*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_{v*}^1 \beta_v dv}, \tag{32}$$

respectively. We can invoke Proposition 4 to claim that the world equilibrium will either feature (i) an indeterminate pattern of trade and factor price equalization; or (ii) complete specialization, with the relatively capital abundant country completely specialized in sectors with relatively long $\mathcal{APP}$s (or $v < v^*$), and with this country featuring a lower interest rate and a higher wage rate than the other country even with free trade. But for this particular case, we can further establish (see Appendix A.5):

Proposition 5. *In the Wicksell-Swan's point-input, point-output model with constant-elasticity productivity functions, the world equilibrium exists. If ϕ^H/ϕ^F is sufficiently lower than one, Home completely specializes in sectors with relatively long average production periods ($\mathcal{APP}s$), while factor price equalization is achieved if ϕ^H/ϕ^F is sufficiently close to 1.*

In the left panel of Figure 3, we illustrate this result for the case in which $\beta_v = 1$ for all v and $\zeta_v = 1 - v$.[25] The left panel of the figure plots the boundaries of the FPE set in the space $\left(\phi^H, \phi^F\right)$ for the case in which $L^H = L^F$. The lower boundary associated with $\phi^H < \phi^F$ (our running case) can easily be derived from equations (31) and (32), and is given by

$$\underline{\phi^H/\phi^F} = \frac{e^{v^*} - v^* e}{1 - \left(e^{v^*} - v^* e\right)} \frac{v^*}{1 - v^*}, \tag{33}$$

where v^* is obtained by setting $B\left(v^*\right) = 1$ in equation (30).[26] The upper bound (applying when instead Foreign is more patient and thus $\phi^H > \phi^F$) is simply the reciprocal of $\underline{\phi^H/\phi^F}$. As shown in the figure, factor price equalization is attained as long as $\phi^H/\phi^F \in (0.34, 2.92)$. When $\phi^H/\phi^F < 0.34$, the equilibrium is instead one with complete specialization and Home featuring a lower interest rate, while when $\phi^H/\phi^F > 2.92$, it is Foreign that features a lower interest rate and specializes in long-$\mathcal{APP}$ sectors.

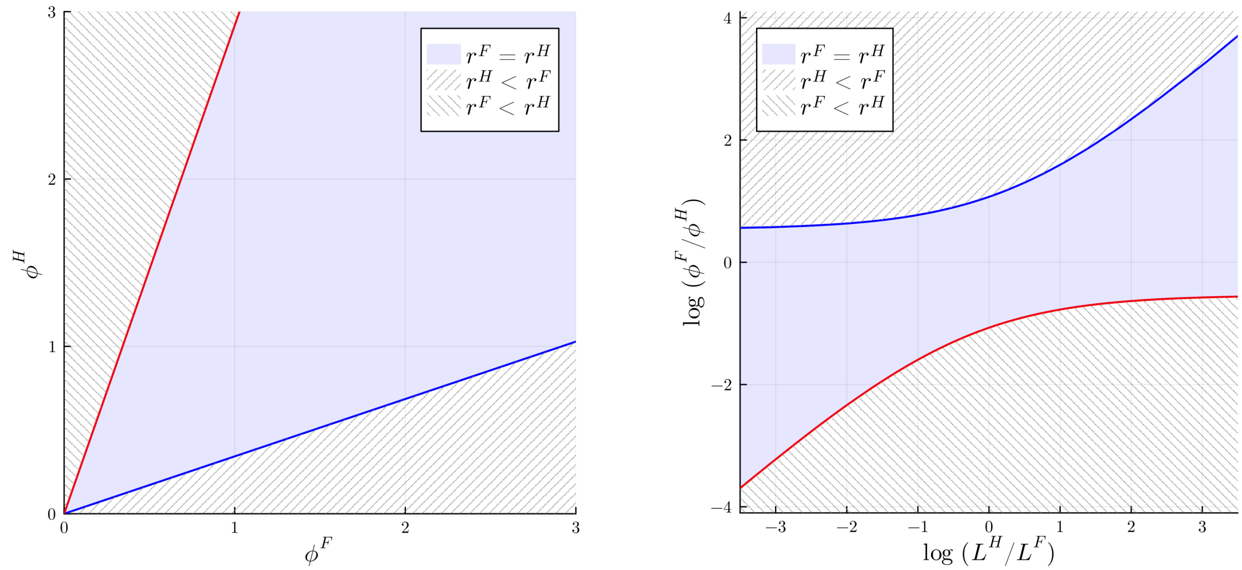
The right panel of Figure 3 generalizes this analysis to the differences in population size across the two countries. On the Y-axis, we plot the inverse of the ratios $\underline{\phi^H/\phi^F}$ and $\overline{\phi^H/\phi^F}$ for various values of L^H/L^F, both in log scale to aid visualization. Notice that as long as ϕ^F/ϕ^H is

[25]For this particular parameterization, we can also show that the equilibrium is unique whenever ϕ^H/ϕ^F is low enough to ensure that factor prices are not equalized (see Appendix A.5).

[26]This delivers $v^* = \ln\left(\left(e+1\right)/2\right) \simeq 0.62$. Equation (33) then delivers $\underline{\phi^H/\phi^F} \simeq 0.34$.

Figure 3: Equilibria in the Wicksell-Swan Model when $\beta_v = 1$ and $\zeta_v = 1 - v$

(a) Equilibria with identical populations **(b)** Equilibria as a function of ρ^H/ρ^F and L^H/L^F



Notes: The figures plot the nature of equilibria as a function of the patience parameters and populations for the special case of Wicksell-Swan economy with $\beta_v = 1$ for all v and $\zeta_v = 1 - v$. The left panel imposes $L^H = L^F$ and plots equilibria in the space (ϕ^H, ϕ^F), while the right panel plots equilibria in the space $(\log(\phi^F/\phi^H), \log(L^H/L^F))$.

sufficiently close to 1 ($\log\left(\phi^F/\phi^H\right) = 0$), factor price equalization is achieved for any relative size L^H/L^F, consistent with Proposition 5.[27]

These results indicate a close resemblance of the equilibrium of the model with that of the Heckscher-Ohlin model, and specifically its version with a continuum of goods in Dornbusch et al. (1980). In that model, factor price equalization is attained whenever differences in capital-labor ratios across countries are small relative to differences in capital intensity across sectors. The region of FPE is often plotted using a (production) Edgeworth box that represents different allocations of worldwide capital and labor across countries. In our model, the stock of capital is not given, but is endogenous to population and to other parameters of the model. This justifies our distinct representation of the range of parameter values for which FPE is attained, but in Online Appendix C.3, we illustrate the FPE set for the variant of our model with a fixed supply of capital.

As pointed out above, another more significant difference relative to the Heckscher-Ohlin model is that we identify differences in the $\mathcal{APP}$ as a key determinant of comparative advantage.

[27] Although we have illustrated the equilibrium of this Wicksell-Swan model with labor only used at the beginning of production, it is straightforward to extend the analysis to the case in which labor is used at a uniform rate along the production process. In such a case, the equilibrium conditions are analogous to those derived above, except that the term $e^{-\zeta_v}$ in the equations above is replaced throughout with $e^{-\tilde{\zeta}_v}$, where $\tilde{\zeta}_v$ is a monotonically increasing transformation of ζ_v.

In the absence of time-versus-capital intensity ranking reversals, these differences correspond to differences in working-capital intensity, but they need not match variation in *physical capital* intensity across sectors. Indeed, the measure of the $\mathcal{APP}$ developed by Antràs and Tubdenov (2025) is *negatively* correlated with physical capital intensity. In Section 5, we will explore this distinction empirically when testing our model.

4.4 Equilibria with Time-Versus-Capital Intensity Ranking Reversals

It is well understood that the results in Proposition 4 may break down in the presence of capital intensity reversals. Similarly, and as mentioned above, the sharp comparative advantage result in Proposition 3 relies on the absence of $\mathcal{APP}$ reversals. In this section, we study the more novel implications of the possible existence of time-versus-capital intensity ranking reversals. Remember from Definition 2 that this reflects situations in which there does not exist a consistent ranking of sectors in terms of time intensity and capital intensity. To isolate the role of this type of reversals, we consider environments with no $\mathcal{APP}$ reversals, no capital intensity reversals, and with capital intensity declining in the interest rate. We will develop a specific example with such features later in this section.

The presence of time-versus-capital intensity ranking reversals leads to novel implications for the nature of the free trade equilibrium—both relative to our results for regular economies and to the standard predictions of the Heckscher-Ohlin model. In the main text, we eschew technical details and instead focus on presenting these unconventional results, offering intuition for them, and illustrating them numerically. Online Appendix C.4 offers a formal exposition.

We emphasize three peculiar aspects of potential equilibria in *irregular* economies featuring time-versus-capital intensity ranking reversals. First, there is the possibility that, for given parameter values (patience rates, monitoring costs, population sizes, etc.), there might exist *multiple equilibria* with qualitatively different patterns of specialization in different equilibria. Second, in some of these equilibria, the relatively capital abundant country might specialize in goods featuring a *lower* capital intensity than the capital intensity with which it would operate the goods it imports. Third, there might exist equilibria in which the relatively capital scarce country might end up with a strictly *lower* interest rate. None of these situations can arise in our *regular economies* or in a Heckscher-Ohlin model featuring no capital intensity reversals. Let us next elaborate on each of these three possibilities.

To build intuition, consider an initial situation in which both countries are fully symmetric, with $\rho^H = \rho^F$, $m^H = m^F$, and $L^H = L^F$. In *regular* economies, such symmetry always results in a factor price equalization equilibrium. With time-versus-capital intensity ranking reversals, such an equilibrium continues to exist, but there is now the possibility of two other type of equilibria with complete specialization. Intuitively, if one of the two countries were to completely specialize in the relatively long-$\mathcal{APP}$ sectors, *and* these sectors happened to feature relatively *low* capital intensity, then this would result in a relatively low demand for capital and a lower interest rate for that country, which in turn would confer this country comparative advantage in those long-$\mathcal{APP}$ sectors. The potential existence of this *positive feedback loop* is the key for the possibility of multiple equilibria in our model. In the Heckscher-Ohlin model, when a country specializes in sectors with low capital intensity, the rental rate of capital also falls. However, this decline disproportionately lowers the relative prices of goods produced in capital-intensive sectors, thereby breaking the above positive feedback loop. The reason for this is that in the Heckscher-Ohlin model, Shephard's lemma creates a close link between relative production costs (or relative prices) and factor intensity, while in our framework such a tight connection does not necessarily hold due to the heterogeneous nature of capital goods.

The existence of multiple equilibria remains possible in a neighborhood of this symmetric starting point. When equilibria entail the more patient country (Home) specializing in long-$\mathcal{APP}$ sectors, as in *regular* economies, then there is the possibility that Home exports goods featuring a lower capital intensity than the capital intensity it would employ to produce the goods it imports, something that can never occur in a Heckscher-Ohlin framework (absent capital intensity reversals). When instead equilibria entail the *less* patient country specializing in the more time-intensive (long-$\mathcal{APP}$) sectors, then this country ends up with a lower interest rate despite being relatively capital scarce, something that again could *not* occur in our *regular economies* or in the Heckscher-Ohlin model.

Despite the potential for multiple equilibria – and even though the more patient country may export goods that are less capital intensive than the goods it imports – it is important to emphasize that the relatively capital abundant country is a net exporter of capital services (and a net importer of labor services) in all equilibria (see Online Appendix C.4). The factor-content implications of our model thus remain aligned with those in Heckscher-Ohlin-Vanek model.

An Illustration: Chopping Off Trees To further illustrate the implications of time-versus-capital intensity ranking reversals, we consider a simple extension of the Wicksell-Swan (timber) model, in which labor is used both at the beginning of production (e.g., to plant seeds) *and* also at the end of production (e.g., to chop off trees). More specifically, suppose that technology is given by the Cobb-Douglas function

$$y_v\left(\boldsymbol{L}_v, T_v\right) = Z_v\left(T_v\right) F_v\left(\boldsymbol{L}_v\right) = \left(T_v\right)^{\zeta_v} \left(\ell_v^0\right)^{\alpha_v} \left(\ell_v^T\right)^{1-\alpha_v}. \tag{34}$$

Notice that we maintain the assumption that productivity $Z_v\left(T_v\right)$ is a constant-elasticity function of the production length T_v, but we now let output use labor both at the beginning of production (e.g., to plant trees in the lumber example) and also at its end (e.g., to chop off the trees at T_v).

The parameter α_v governs the relative importance of period-0 and period-T labor in production.

Given equation (34), firms in sector v then solve the problem:

$$\max_{T_v, \ell_v^0, \ell_v^T} \pi^j = p_v^j \left(T_v\right)^{\zeta_v} \left(\ell_v^0\right)^{\alpha_v} \left(\ell_v^T\right)^{1-\alpha_v} e^{-r^j T_v} - w^j \ell_v^0 - w^j \ell_v^T e^{-r^j T_v}.$$

Straightforward manipulations of the first-order conditions of this problem deliver an optimal production length equal to $T_v^j = \zeta_v / \left(\alpha_v r^j\right)$, and an associated average period of production equal to $\mathcal{APP}_v = \zeta_v / r^j$.

As in Wicksell's lumber model, the cross-section of $\mathcal{APP}$s is shaped by the vector $\boldsymbol{\zeta} = \{\zeta_1, \zeta_2, ..., \zeta_v\}$, and we can again order sectors such that ζ_v is decreasing in v, with no possibility of $\mathcal{APP}$ reversals. Indeed, the zero-profit condition can be shown to imply that

$$p_v^j = \kappa_v w^j \left(r^j\right)^{\zeta_v},$$

for some constant κ_v, thus highlighting the importance of the interaction of the vector $\boldsymbol{\zeta}$ and of interest rate differentials in shaping specialization. Capital intensity in sector v is in turn given by

$$\frac{K_v^j}{L_v^j} = \frac{w^j}{r^j} \frac{\alpha_v \left(e^{\zeta_v/\alpha_v} - 1\right)}{\left(1 - \alpha_v\right) e^{\zeta_v/\alpha_v} + \alpha_v}, \tag{35}$$

and necessarily decreases in the interest rate r^j.[28] It is straightforward to see that there *cannot* be capital intensity reversals whenever K_v^j/L_v^j is given by equation (35).

Nevertheless, for certain values of the vector $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, ..., \alpha_v\}$, there can certainly exist time-versus-capital intensity ranking reversals, in the sense that K_v^j/L_v^j need not always be higher for sectors with a lower index v (and thus longer $\mathcal{APP}$s).[29] Intuitively, though sectors with a higher time intensity feature longer $\mathcal{APP}$s regardless of the relative importance of the labor used to initiate or complete the production process, the demand for working capital is shaped by both time intensity *and* by the relative importance of the labor used to initiate the process, as proxied by the vector $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, ..., \alpha_v\}$.[30]

What are the consequences of the possibility of time-versus-capital intensity ranking reversals? As long as Home continues to feature a lower interest rate than Foreign in the trade equilibrium,

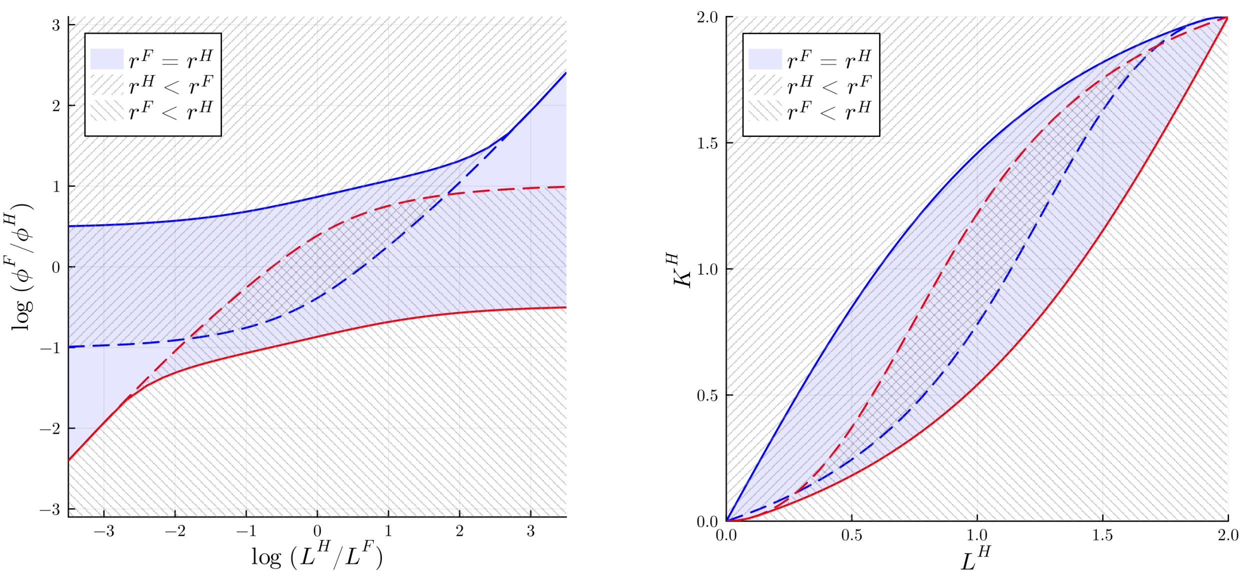
[28]To derive equation (35), note that $K_v^j = \int_0^{T_v} w^j \ell_v^{0j}(t) \frac{e^{r^j(T_v - t)} - 1}{r^j} dt$ and $\ell_v^{Tj}(t) = \frac{1-\alpha_v}{\alpha_v} e^{rT_v} \ell_v^{0j}$.

[29]For instance, if $\zeta_v = 1 - v$ and $\alpha_v = e^{-(1-v)}$, K_v^j/L_v^j is only decreasing in v for values of v above (roughly) 0.266.

[30]Equation (34) implicitly assumes that lengthening the interval of production increases gross output rather than merely the value added of the initial labor input. The existence of time-versus-capital intensity ranking reversals continues to be possible when output is instead given by $y_v\left(\boldsymbol{L}_v, T_v\right) = \left(\left(T_v\right)^{\zeta_v} \ell_v^0\right)^{\alpha_v} \left(\ell_v^T\right)^{1-\alpha_v}$. In that case, the $\mathcal{APP}$ is given by $\mathcal{APP}_v = \alpha_v \zeta_v / r^j$, while capital intensity is still given by equation (35). It is then easy to construct examples in which the $\mathcal{APP}_v$ is still monotonically decreasing in v, while K_v^j/L_v^j is not. One such example is the same one mentioned in footnote 29, with $\zeta_v = 1 - v$ and $\alpha_v = e^{-(1-v)}$.

Figure 4: Equilibria in the Wicksell-Swan Model when Chopping Off Trees

(a) Endogenous Capital Supply **(b)** Fixed Capital Supply



Notes: The figures plot the nature of equilibria in the Wicksell-Swan economy with Chopping Off Trees when $\beta_v = 1$, $\alpha_v = e^{-3(1-v)}$ and $\zeta_v = 1 - v$. The left panel plots equilibria with endogenous capital supply, while the right panel shows equilibria when capital supply is fixed.

the pattern of specialization will remain the same as summarized in Proposition 3 and part (a) of Proposition 4 will continue to hold, with Home producing all goods with $v < v^*$. Provided that ϕ^F is sufficiently higher than ϕ^H, this will remain the unique type of equilibrium in the model because equations (28) and (29) will deliver a lower interest rate at Home regardless of the relative demand for capital in both countries.

For smaller differences in patience across countries, however, there arises the possibility of an alternative equilibrium in which Foreign specializes in low-v sectors, if these sectors also happen to feature low capital intensities, thereby allowing Foreign to sustain a lower interest rate. Naturally, in those situations, there exists an alternative equilibrium in which it is Home that specializes in these low capital-intensity sectors, thereby putting further downward pressure on their interest rate relative to the Foreign one. Finally, close to the symmetric situation with $\phi^H = \phi^F$, there remains the possibility of an equilibrium with factor price equalization.

We illustrate these results in the left-panel of Figure 4 for the same parametric case ($\beta_v = 1$ for all v and $\zeta_v = 1 - v$) we developed in Section 4.3, but with $\alpha_v = e^{-3(1-v)}$. In that case, capital intensity is *increasing* for all $v < 0.625$, while the $\mathcal{APP}$ is decreasing in v. Comparing this left panel to the right-panel of Figure 3, we see that as long as ϕ^F/ϕ^H is sufficiently high, the unique equilibrium is one with Home featuring a lower interest rate, and thus specializing in the long-$\mathcal{APP}$ sectors. Similarly, if Foreign were to be much more patient or financially developed (so ϕ^F/ϕ^H is sufficiently low), then the unique equilibrium would be one with Foreign featuring a lower interest rate and exporting the goods with relatively long production processes. For values of ϕ^F/ϕ^H, we see the emergence of an equilibrium with factor price equalization (the shaded area), but unlike in Figure 3, notice that for certain parameter values, equilibria with complete specialization and interest rate differentials may also exist. Around the full symmetry point, all three types of equilibria are possible.[31]

The right-panel of Figure 4 illustrates the multiplicity of equilibria in the variant of our model with fixed capital supply (see Online Appendix C.3 for details). In such a case, we can do so with the more familiar production Edgeworth box representing different allocations of worldwide capital and labor across countries. An equilibrium with FPE always exists around the 45-degree line, but we observe the existence of multiple equilibria – some involving two equilibria and other three equilibria – for most allocations within the FPE set.

5 Empirical Evidence

In this section, we explore the empirical relevance of the temporal structure of production in shaping international specialization. We test a weak version of Proposition 3, namely whether countries with lower cost of (financial) capital tend to export *disproportionately* more in sectors with longer average production periods. We proceed along the lines of the empirical test of the Heckscher-Ohlin model in Romalis (2004), and the subsequent literature on the institutional determinants of comparative advantage pioneered by Nunn (2007), Levchenko (2007), Costinot (2009b), and Manova (2013). More specifically, we regress country-industry level exports on interactions of country-level and industry-level characteristics, while controlling for common industry and country trends.

To be more precise, we are interested in specifications of the type

$$\log\left(\text{Exports}_{j,v,t}\right) = \delta_{\mathcal{APP}} \times \left(\mathcal{APP}_v \times r_{j,t}\right) + \boldsymbol{\delta}_X \times \left(\boldsymbol{X}_v \times \boldsymbol{X}_{j,t}\right) + \mu_{v,t} + \mu_{j,t} + \varepsilon_{j,v,t}, \tag{36}$$

where $\log\left(\text{Exports}_{j,v,t}\right)$ are log exports of country j in sector v in year t, $\mathcal{APP}_v$ is the average period of production of sector v, $r_{j,t}$ is a measure of the cost of capital in country j in year t, $\boldsymbol{X}_v \times \boldsymbol{X}_{j,t}$ is a vector of interactions of industry- and country-level characteristics, $\mu_{v,t}$ are industry-year fixed effects, $\mu_{j,t}$ are country-year fixed effects, and $\varepsilon_{j,v,t}$ is an error term.

Our key interaction term is $\mathcal{APP}_v \times r_{j,t}$ and is constructed as the product of a recently proposed sectoral measure of the $\mathcal{APP}$ developed by Antràs and Tubdenov (2025) and a

[31] We should emphasize that it is certainly possible for multiple equilibria to arise *only* when countries are asymmetric. If time-versus-capital intensity ranking reversals are moderate, FPE may be the unique equilibrium when countries are symmetric, but multiple equilibria may still exist in other areas of the FPE set in which countries are asymmetric. We provide an example of this in Online Appendix C.4.

country's level of financial development (as measured in Bilir et al., 2019) – a proxy for the cost of borrowing $r_{j,t}$. More specifically, Antràs and Tubdenov (2025) show that, under stationarity conditions, the $\mathcal{APP}$ of a firm can be approximated by the ratio of a firm's stock of inventories to the cost of the goods it sells (or COGS). They then compute the $\mathcal{APP}$ using data from publicly traded companies worldwide, and construct country-industry level measures. Consistent with our model, Antràs and Tubdenov (2025) find that the $\mathcal{APP}$ tends to increase as the cost of capital faced by firms decreases. Furthermore, cross-country comparisons reveal that $\mathcal{APP}$ reversals do *not* tend to be pervasive, thus lending credibility to the assumption underlying Proposition 3. To maximize industry coverage and alleviate endogeneity concerns, we employ their measure of the $\mathcal{APP}_v$ computed only with data from US firms. With regard to the proxy for the cost of capital $r_{j,t}$, we refrain from using a direct measure of the cost of capital, again with endogeneity concerns in mind. Instead we use a proxy $r_{j,t}$ based on variation in the supply of capital emanating from differences in the depth of capital markets (see Appendix B for more details).

The vector of controls $\boldsymbol{X}_v \times \boldsymbol{X}_{j,t}$ in equation (36) includes various interaction terms. Two of these terms are motivated by the Heckscher-Ohlin model: first, a sector's physical capital-labor ratio interacted with a country's physical capital-labor endowment (labelled as $K_v/L_v \times K_{j,t}/L_{j,t}$ in Table 1), and second, a sector's relative use of skilled labor interacted with a country's relative endowment of skilled workers (labelled as $H_v/L_v \times H_{j,t}/L_{j,t}$ in Table 1). A third interaction control is motivated by the work of Rajan and Zingales (1998) and Manova (2013) and addresses the concern that the time intensity of production might be correlated with dependence on outside financing. With that in mind, our regressions include an interaction between a sector's dependence on outside financing and a country's level of financial development (labelled as $EFD_v \times FD_{j,t}$). Appendix B contains details on how each of these interaction variables are constructed.

Table 1 reports our estimates. All variables are standardized to have mean zero and standard deviation one, so the interpretation of column (1) is that a one standard deviation increase in the interaction $\mathcal{APP}_v \times r_{j,t}$ raises log exports by 0.037 standard deviations. This result validates our model's prediction that countries with lower costs of capital (due to deeper financial markets) appear to feature disproportionately higher exports in time intensive (long-$\mathcal{APP}$) sectors, even when controlling for country-year and industry-year fixed effects. In column (2), we show that this result is robust to including the three control interactions mentioned above. Consistent with the results in Romalis (2004), the Heckscher-Ohlin prediction in column (2) is better borne by the skilled-labor interaction than by the physical-capital interaction, with the latter actually having an insignificant impact on exports. Comparing the relative magnitude of the interaction terms coefficients, the comparative advantage afforded to financially developed countries in long-$\mathcal{APP}_v$ sectors is on par with that given to skill endowed countries in skill intensive sectors.

Table 1: Comparative Advantage

	log Exports$_{v,j,t}$				
	(1)	(2)	(3)	(4)	(5)
$\mathcal{APP}_v \times FD_{j,t}$	0.037***	0.025***	0.045***	0.050***	-0.006
	(0.007)	(0.006)	(0.008)	(0.008)	(0.005)
$K_v/L_v \times K_{j,t}/L_{j,t}$		0.015	0.000	0.006	0.019
		(0.027)	(0.027)	(0.062)	(0.038)
$H_v/L_v \times H_{j,t}/L_{j,t}$		0.026**	0.037***	-0.114***	0.207***
		(0.010)	(0.011)	(0.015)	(0.019)
$EFD_v \times FD_{j,t}$		0.016***	0.016***	0.002	0.017***
		(0.004)	(0.004)	(0.005)	(0.003)
$\mathcal{APP}_v \times K_{j,t}/L_{j,t}$			-0.127***		
			(0.037)		
$\mathcal{APP}_v \times H_{j,t}/L_{j,t}$			0.003		
			(0.016)		
Observations	472,023	472,023	472,023	472,023	472,023
R^2	0.79	0.79	0.79	0.80	0.91
Country-Year fixed effects	✓	✓	✓	✓	✓
Industry-Year fixed effects	✓	✓	✓	✓	
Country-Industry fixed effects					✓
$GDP_{j,t}$ × Industry fixed-effect				✓	

Note: ***: 0.01, **: 0.05, *: 0.1. Standard errors clustered at the country-industry level. Standardized beta coefficients are reported. The dependent variable, $\log \text{Exports}_{v,j,t}$, has a standard deviation of 4.00 in the sample.

And, it is larger than the comparative advantage that financially developed countries get in sectors which rely on outside financing.

To give some substance to the magnitude of our estimates, consider trade in Pharmaceutical Preparation Manufacturing, for which world exports totaled $428 billion in 2019. This industry features one of the longest production lengths as measured by $\mathcal{APP}_v$ (see Table 1 in Antràs and Tubdenov, 2025). And recently, it has seen increased offshoring to low cost countries, namely China and India. Assuming our estimates are causal, if India saw its financial development (as measured by $FD_{j,t}$) rise to the level of China's, then its exports would increase by 48% or $8.3 billion. And, its share of world exports would rise from 4% to 6%.[32] For comparison, if India attained the relative skill endowment of China, its exports would rise by only 7%.

In columns (3) and (4) we attempt to control for confounders that might generate a spurious dependence between time intensity and financial development. Namely, financial development may be correlated with other features of development like capital or skill accumulation, which

[32] This example features $\mathcal{APP}_v = 0.43$, $FD_{IND,2019} = 51$, and $FD_{CHN,2019} = 165$. The interaction $\mathcal{APP}_v \times FD_{j,t}$ has a standard deviation of 12.39, so the difference $\mathcal{APP}_v \times (FD_{CHN,2019} - FD_{IND,2019})$ represents a change of 3.94 standard deviations. The standard deviation of log exports is 4.00, so using the estimates from column (2) of Table 1, we have a change in exports of $100 \times (\exp(3.94 \times 0.025 \times 4.00) - 1) = 48\%$.

may otherwise be important for exporting in long-$\mathcal{APP}_v$ sectors.[33] Column (3) allows additional interactions between $\mathcal{APP}_v$ and country-level characteristics, and column (4) controls for GDP-per-capita trends that are industry-specific. The economic magnitude of our key interaction is actually strengthened, while the other interactions turn insignificant or of the wrong sign.

Finally, we develop a complementary test of the determinants of comparative advantage by examining time series variation in exporting within country-industry pairs. Namely, we test if countries which grow more in a particular characteristic, also export more in the industries that rely on those characteristics intensely. Econometrically, this boils down to replacing the industry-year fixed effects in columns (1) through (4) with country-industry fixed effects. Column (5) implements this test and delivers an effect of our interaction that is indistinguishable from zero. We are however not disheartened by this result because our model is one characterizing steady states and not one capturing how yearly changes in factor endowments shape specialization in the short run. This same caveat of course also applies to more traditional models of comparative advantage, so it is perhaps not surprising that previous empirical tests of comparative advantage (see Chor, 2010, or the survey in Nunn and Trefler, 2014) rarely exploit panel data, focusing instead on cross-sectional tests.

Appendix B provides further robustness checks. We examine additional interactions between the country-level and industry-level characteristics, consider lagged regressors to address simultaneity, and present estimates under various sample restrictions. Overall, our results are admittedly far from conclusive, but they are highly suggestive that comparative advantage seems to be shaped by the temporal aspects of production emphasized in this paper, and in a manner consistent with our model.

6 Extensions

In this last section, we outline several extensions of our framework. Given space constraints, we relegate many details to the Online Appendix.

6.1 Costly Trade

We first relax the assumption that international trade in finished goods does not entail trade costs. We begin by considering the canonical formulation of trade costs as being 'instantaneous' iceberg costs proportional to the value of the good being transacted. We next leverage the explicit treatment of time in our model to study the effect of trade costs associated with the

[33] Antràs and Tubdenov (2025) find that the $\mathcal{APP}$ is positively correlated with skill intensity, possibly reflecting the disproportionate need for care and precision in skill-intensive processes, while it is *negatively* correlated with physical capital intensity.

time it takes to ship goods across borders, and show how world interest rates shape the pattern of specialization.

Standard Iceberg Trade Costs To introduce costly trade in the simplest possible way, we begin by assuming that in shipping between Home and Foreign, a fraction of goods melts in transit, so τ units of a good need to be shipped for 1 unit to make it to the other country. For the time being, we assume goods are received in the same instant they are shipped, as with digital trade.

The introduction of trade costs has no impact on the cost minimization problem in (3), which delivers the same path of unit labor requirements, and the same $\mathcal{APP}$ (see Online Appendix C.5). The main novelty is that, when selling to the foreign market, this unit cost in (4) is multiplied by a factor $\tau > 1$ to obtain the minimum price at which producers are willing to sell abroad. Nevertheless, Proposition 1 continues to hold, and thus comparative advantage is still shaped by the interaction of $\mathcal{APP}$ differences across sectors and cost-of-capital differences across countries.

There are three main implications of standard iceberg trade costs for the pattern of specialization. First, as is well known, the set of parameters for which factor price equalization is attained becomes a set of measure zero (see Helpman and Krugman, 1985). As a result, the equilibrium is generically one with one country featuring a lower interest rate and specializing in relatively long-$\mathcal{APP}$ sectors, in line with Proposition 3.[34] Second, and as in the Ricardian model in Dornbusch et al. (1977), trade costs lead to the emergence of an endogenous set of nontraded goods. The third prediction is more tightly related to the novel aspects of our framework and concerns the variant of our model with an endogenous production length. More specifically, it is straightforward to show that the optimal production length is not only identical under autarky and free trade (see footnote 24), but it is actually *independent* of the level of trade costs τ. The reason for this is that the marginal benefit of letting production mature is associated with a reduction in marginal costs, while the marginal cost of letting production mature is associated with delaying the collection of revenue. Because production costs and revenue are proportional to each other, the choice is independent of scale (and of trade costs). In sum, the temporal aspects of production (the path of labor use and the optimal production length) are independent of trade costs.[35]

Time as a Trade Cost An advantage of having developed a framework with an explicit notion of production time is that we can apply the same machinery to introduce a temporal

[34]Whether such an equilibrium is unique or not depends again on the existence or time-versus-capital intensity ranking reversals.

[35]Antràs (2023b) shows that this result remains true in monopolistically competitive environments with constant markups.

dimension of trade costs. Specifically, we now assume that, relative to domestic transactions, shipping goods across borders involves an additional interval of time d between the time at which production is completed and the time at which the shipment is received and payment is made. In terms of standard terminology in trade finance, we thus assume that all international transactions are carried out on 'open account' terms (i.e., involving post-shipment payments). We also continue to assume that shipping involves iceberg trade costs τ (shipping fees, insurance, etc.) incurred after production but before the shipment arrives at the importer's country.[36]

As in the case of iceberg trade costs, the presence of temporal trade costs turns out to be immaterial for the path of unit labor requirements and for the length of *production* processes (see Online Appendix C.5).[37] As a result, the unit cost of production $c(w, r)$ in equation (4) is multiplied by a factor $\tilde{\tau}^j \equiv \tau e^{r^j d} > 1$ to compute the minimum price at which producers in country j are willing to sell goods in country $j' \neq j$. It is clear that some of the implications of these temporal trade costs are analogous to those resulting from (instantaneous) iceberg trade costs. The FPE set becomes of measure zero, the general equilibrium will feature a range of nontraded goods, and the optimal length of production is independent of d. Nevertheless, the modeling of trade costs as an interval of time between production and delivery generates at least two additional implications.

First, because the equilibrium is generically one without factor price equalization, we necessarily have that either $\tilde{\tau}^H < \tilde{\tau}^F$ or $\tilde{\tau}^H > \tilde{\tau}^F$ and thus trade costs are asymmetric and relatively lower in the country featuring a lower interest rate. This implication is in line with the results in Waugh (2010), who shows that to reconcile bilateral trade volumes and price data within a standard gravity model, trade frictions between rich and poor countries must be systematically asymmetric, with poor countries facing higher costs to export relative to rich countries.[38]

Second, because trade costs are now partly shaped by the time value of money, reductions in interest rates worldwide tend to reduce trade costs. As a result, a potential contributor to the growth in world trade in the late 1980s, 1990s and 2000s may have been the secular decline in interest rates during that period (as documented, for instance, by Barkai, 2020).

[36] In our model, the only temporal cost of shipping is the interest cumulated over the interval of time d. In practice, one could imagine that the value of goods may be diminished while in transit if these goods are perishable or if their demand is time sensitive. Conversely, some goods, when properly shipped, may mature in transit and their value may be enhanced by the interval d. We leave these extensions for future work.

[37] Although the $\mathcal{APP}$ is common for domestic transactions and for exports, naturally the collection of export revenue is delayed by an additional time interval d.

[38] Antràs (2015) further shows that these asymmetries are not only correlated with income per capita differences, but also with measures of the quality of contracting institutions, which tend to be negatively correlated with interest rates across countries (see Djankov et al., 2007).

6.2 Global Value Chains

In our baseline model, all goods are produced with local labor and only final goods are traded. In this section, we consider a variant of our model in which we allow for trade in semi-finished goods. More specifically, we envision an environment in which production can be broken down into a number of sequential stages, with each stage combining labor with the good produced up to the prior stage, and in which these semi-finished stages can be traded internationally at the end of each stage. Building on the analogy of Wicksell's point-input, output-model, lumber is not typically sold to final consumers, but to downstream producers that may use it to produce (tradable) wood panels, which may in turn be employed by furniture makers to make (tradable) desks available for final consumers and for downstream firms that use desks in the production of other types of (tradable) consumer goods.

Set-Up This variant of our model is closest to the framework developed in Antràs and de Gortari (2020) and makes a number of simplifying assumptions. We assume that consumers now value a single final good, which is taken to be the numéraire. This final good needs to undergo N stages in a pre-determined, sequential order. The last stage of production can be interpreted as final-good assembly and is indexed by N. We focus on the case in which the number of stages is discrete (so N is an integer number), but the analysis can be extended to the case of a continuum of stages without difficulty. At each stage $n > 1$, production combines labor with the good finished up to the previous stage $n - 1$. Production in the initial stage $n = 1$ only uses labor, and production technologies in all stages are homogeneous of degree one and are given by

$$y_n\left(\boldsymbol{L}_n, \mathbf{y}_{n-1}\right) = \left(Z_n\left(T_n\right) F_n\left(\boldsymbol{L}_n\right)\right)^{\alpha_n}\left(y_{n-1}\right)^{1-\alpha_n}, \tag{37}$$

where y_n is output obtained at the end of stage n. In this expression, Z_n denotes labor productivity at stage n (potentially a function of the length T_n of production), and $\alpha_n \in [0, 1]$ is labor or value-added intensity at stage n, with $\alpha_1 = 1$. As in our baseline model, $\boldsymbol{L}_n$ is an infinite-dimensional vector of labor use along the production process, i.e., $\boldsymbol{L}_n = \{\ell_n(t)\}_{t \in [0,T_n]}$. We assume that the stage $n - 1$ output is purchased at the beginning of production, and then is transformed with labor applied over an interval $t \in [0, T_n]$ into stage n output, but our results are robust to a case in which production at stage n uses stage $n - 1$ output at various instants within $t \in [0, T_n]$ (see Online Appendix C.6).[39]

The technologies in equation (37) are freely available to all agents in the economy, and the various stage-specific goods are produced under perfect competition.

[39] Our main results also continue to apply if $Z_n(T_n)$ augments gross output rather than the value added of labor, i.e., $y_n = Z_n(T_n)\left(F_n\left(\boldsymbol{L}_n\right)\right)^{\alpha_n}\left(y_{n-1}\right)^{1-\alpha_n}$.

Free Trade Equilibrium In Online Appendix C.6, we study the autarky and free trade equilibria of this model of global value chains. Despite the sequential nature of production, most features of equilibrium are qualitatively similar to those of our model with a continuum of (horizontally) differentiated sectors. Most notably, comparative advantage is again shaped by the interaction of interest rate differences across countries and $\mathcal{APP}$ differences across stages. This is due to the fact that we can again show that

$$\frac{\partial \ln c_n^j}{\partial r^j} = \mathcal{APP}_n^j,$$

where c_n^j is the unit cost of production of stage n in country j, and where $\mathcal{APP}_n^j$ is a generalized measure of the average period of production that takes into account the timing of both labor *and* stage $n-1$ input expenditures (see Online Appendix C.6). An important caveat, however, is that comparative advantage does not only depend on the interaction of the $\mathcal{APP}$ of a sector and interest rate differentials across countries, but also on the interaction of cross-sectoral variation in labor- or value-added intensity (α_n) and wage differentials across countries. The reason for this is that stage-n prices are no longer linear in wages, but are rather a Cobb-Douglas of wages and of the price of stage$-n-1$ output. Nevertheless, in Online Appendix C.6, we show that a variant of Proposition 3 continues to hold in this world with GVCs, and the empirical evidence described in Section 5 is also consistent with this variant of our model.

Because comparative advantage is driven by the interaction of stage-specific $\mathcal{APP}$s and interest rates, whether low interest countries have comparative advantage in a specific stage of production turns out to be independent of the relative upstreamness of that stage. This is despite the fact that the allocation of labor to the various stages *is* shaped by the positioning of that stage: other things equal, less labor is allocated to relatively upstream stages of production because the effective cost of labor (inclusive of cumulated interest payments) is higher for those upstream stages (see Online Appendix C.6). Despite some subtle differences, the equilibrium of the model in the presence of factor price differences is akin to that of our multi-product Ricardian model with horizontally differentiated goods.[40] More specifically, a variant of Proposition 4 continues to hold in *regular* economies, while in the presence of a certain type of time-versus-capital intensity ranking reversals, there is the possibility of multiple equilibria and of equilibria with non-standard features (as described in Section 4.4).

It is also straightforward to study the endogenous choice of production length T_n and how it is shaped by interest rates in this variant of the model with GVCs. Indeed, in a previous version of the paper (Antràs, 2023a), we developed an extension of the two-stage Wicksell point-input,

[40] In Online Appendix C.6, we show that the aggregate average period of production in an economy with multi-stage production is higher than the $\mathcal{APP}$ of the various stages of production. As a result, the negative impact of the interest rate on log wages (given by equation (10) in our baseline model) and aggregate capital demand are both higher in a world of sequential production.

point-output model in Findlay (1978) to an environment with multiple stages of production. We outline such a version of the model in Online Appendix C.6.

Costly-Trade Equilibrium In Online Appendix C.7, we also extend the above GVC framework to an environment with positive trade costs. The implications for the pattern of trade are much more complex than in our baseline model with 'horizontal' sectors. As in previous work by Harms et al. (2012) and Baldwin and Venables (2013), we show that trade costs introduce a tendency to 'bunch' contiguous production stages in the same location, in some cases breaking the tight link between comparative advantage and differences in the $\mathcal{APP}$ across sectors.[41] An interesting implication of this result is that reductions in trade costs lead to an 'unbundling' of production processes and to increases in GVC participation. Furthermore, when considering the fact that trade costs are partly associated with the cost of time associated with shipping goods across borders, we show that a reduction in interest rates worldwide tends to lead to an increased slicing of GVCs across countries. As a result, the observed decline in interest rates during the late 1980s, 1990s and 2000s (Barkai, 2020) may have been a contributing force to the significant growth in GVC activity worldwide.

6.3 International Capital-Market Integration

The analysis in the previous sections assumed that capital markets are perfectly integrated within countries in the sense that any producer in country j could borrow at a competitive interest rate r^j. At the same time, we assumed that financial markets were completely segmented across borders since we did not allow for international borrowing and lending, effectively assuming that collecting interest from foreign borrowers involved prohibitively high monitoring costs. In this section, we relax this assumption.

Consider first the possibility of borrowing and lending across countries. In the absence of asymmetries in monitoring costs within or across countries, it is clear that arbitrage will ensure that all borrowers face the same interest rate $\bar{r}$ worldwide, even in the absence of integration in goods markets. Intuitively, producers in the high interest rate country (Foreign) will borrow from savers at Home, and the supply of capital available to producers in both countries will adjust to ensure interest rate equalization. This equalization of interest rates in turn implies the equalization of wage rates, and the equilibrium is akin to one with factor price equalization (given some distribution of available capital in each country). Naturally, and as in Mundell (1957), if trade integration already brings about factor price equalization, there is no incentive for capital to flow across countries.

[41] We adopt the dynamic programming approach developed in Antràs and de Gortari (2020) to demonstrate this result.

Next consider the case in which international lending is associated with disproportionately higher monitoring costs. As before, when financing producers in j, lenders in j incur monitoring costs m^j. We assume that Home is both more patient ($\rho^H < \rho^F$) and more financially developed ($m^H < m^F$), so under autarky, we necessarily have $r^H < r^F$. We now further assume that when lending to producers in $j' \neq j$, lenders incur a monitoring cost $m^{jj'}$ in addition to the domestic cost m^j. We assume that this monitoring cost is bilaterally symmetric, so we can simply define $\mu = m^{HF} = m^{FH}$. If r^H and r^F are the prevailing interest rates without capital flows, then if $r^H - \mu < r^F < r^H + \mu$, the equilibrium will feature *no* capital flows and will continue to be as described in Section 6.3.

The case in which with trade but without capital mobility we have $r^F > r^H + \mu$ is more interesting. In such a scenario, Foreign producers can reduce their financial costs by borrowing from Home savers. Indeed, in that case, capital market integration will lead to $r^F = r^H + \mu$. As a result, interest rates faced by producers at Home and in Foreign will converge but will *not* equalize even with capital mobility, and the pattern of comparative advantage will be shaped by Proposition 1. If capital mobility precedes trade integration, then the above is the unique equilibrium. Nevertheless, if capital account liberalization occurs *after* trade integration, other equilibria are possible in the presence of time-versus-capital intensity ranking reversals. More specifically, if the trade equilibrium is one with $r^F < r^H$ (despite Home's higher patience and financial development), then either capital will not move (if $r^H - \mu < r^F$) or it will flow from Foreign to Home up to the point at which $r^H - \mu = r^F$, with Foreign specializing in time-intensive sectors.

In a previous version of the paper (Antràs, 2023a), we also considered environments in which cross-border monitoring costs may be shaped by the identity of lenders and borrowers. In particular, the literature on trade credit has emphasized that buyers and sellers may be in a better position than 'outside' lenders in monitoring each other (see Petersen and Rajan, 2015, and references therein).[42] In our extended framework with global value chains, this raises the possibility that (i) importers in low interest rate countries may pay for goods before receiving them from exporters in high interest rate countries; and (ii) exporters in low interest rate countries may accept a delay in payment when selling to importers in high interest countries. Depending on when these loans are extended and collected, this gives rise to trade credit (when the loan covers the period of production of the high-interest buyer or seller) or to trade finance (when the loan only covers the period over which goods are in transit). In either case, these types of international borrowing and lending constitute a form of partial capital-market integration, with similar implications to those described above, but with some more subtle

[42]The quantitative relevance of trade credit and trade finance has been well established in the literature (see, among many others, Burkart and Ellingsen, 2004, Klapper et al., 2011, Jacobson and Von Schedvin, 2015, and Antràs and Foley, 2015).

effects associated with the fact that international capital flows are disproportionately larger for international buyer-seller relationships. We refer the reader to Antràs (2023a) for details of this analysis.

7 Conclusion

This paper has developed an 'Austrian' model of international specialization in which the temporal dimension of production emerges as a central determinant of comparative advantage. By embedding Böhm-Bawerk's concept of the average period of production ($\mathcal{APP}$) into a multi-sector Ricardian framework with endogenous interest rates, we have shown that countries with lower autarkic interest rates systematically specialize in sectors characterized by longer production durations. We have discussed the connection of our framework to the Heckscher-Ohlin model and we have proved its tractability across a range of extensions, including iceberg and temporal trade costs, global value chains, and international capital flows. At the same time, the framework uncovers novel mechanisms – most notably, time-versus-capital intensity ranking reversals – that can generate multiple equilibria and unconventional specialization patterns.

Empirically, we test the model's core prediction using a recently developed sectoral measure of the $\mathcal{APP}$. We find that financial development, a proxy for lower capital costs, significantly increases exports in time-intensive sectors, even when controlling for traditional comparative advantage proxies. The $\mathcal{APP}$–financial development interaction variable exhibits comparable or greater explanatory power than canonical Heckscher-Ohlin variables. These findings suggest that the temporal dimension of production and its interaction with capital markets should be incorporated more systematically into models of international trade.

Although we have intentionally kept the model stylized, we are confident that the model could be extended in a variety of ways, perhaps making it more amenable to further empirical and quantitative investigations of the role of *time* in international trade and in other fields in Economics. Within international trade, a particularly natural extension of the model would be to incorporate scale economies and imperfect competition. These are standard features of state-of-the-art models of international trade, and they may interact with the forces in this paper in interesting ways because scale economies are often associated with significant investments by firms, with these investments often being fixed in nature and incurred before any revenue is obtained. More generally, we hope that our work will find broader applications in other fields, thereby delivering further insights on the interplay between the temporal dimension of production, financial conditions, and industrial structure.

References

Alessandria, George, Joseph P Kaboski, and Virgiliu Midrigan, "US Trade and Inventory Dynamics," *American Economic Review*, 2011, *101* (3), 303–07.

Antràs, Pol, *Global Production: Firms, Contracts, and Trade Structure*, Princeton University Press, 2015.

Antràs, Pol, "An 'Austrian' Model of Global Value Chains," NBER Working Paper 30901, January 2023.

_ , "Interest Rates and World Trade: An "Austrian" Perspective," *American Economic Association Papers and Proceedings*, May 2023, *113*, 65–69.

Antràs, Pol and Alonso de Gortari, "On the Geography of Global Value Chains," *Econometrica*, 2020, *88* (4), 1553–1598.

_ **and C Fritz Foley**, "Poultry in Motion: A Study of International Trade Finance Practices," *Journal of Political Economy*, 2015, *123* (4), 853–901.

Antràs, Pol and Vitalii Tubdenov, "Measuring the Average Period of Production," *American Economic Association Papers and Proceedings*, May 2025, *115.*

Baldwin, Richard and Anthony Venables, "Spiders and Snakes: Offshoring and Agglomeration in the Global Economy," *Journal of International Economics*, 2013, *90* (2), 245–254.

Bardhan, Pranab Kumar, "International trade theory in a vintage-capital model," *Econometrica*, 1966, *34* (4), 756–767.

Barkai, Simcha, "Declining Labor and Capital Shares," *The Journal of Finance*, 2020, *75* (5), 2421–2463.

Becker, Randy A., Wayne B. Gray, and Jordan Marvakov, *NBER-CES Manufacturing Industry Database (1958-2018, version 2021a)* 2021.

Bilir, L. Kamran, Davin Chor, and Kalina Manova, "Host-country financial development and multinational activity," *European Economic Review*, 2019, *115*, 192–220.

Böhm-Bawerk, Eugen, *The Positive Theory of Capital*, G. E. Stechert & Co., New York (1930 English translation), 1889.

Bruno, Valentina, Se-Jik Kim, and Hyun Song Shin, "Exchange Rates and the Working Capital Channel of Trade Fluctuations," *American Economic Review Papers and Proceedings*, 2018, *108*, 531–536.

Burkart, Mike and Tore Ellingsen, "In-Kind Finance: A Theory of Trade Credit," *American Economic Review*, 2004, *94* (3), 569–590.

Caballero, Ricardo J, Emmanuel Farhi, and Pierre-Olivier Gourinchas, "An Equilibrium Model of Global Imbalances and Low Interest Rates," *American Economic Review*, 2008, *98* (1), 358–93.

Carreras-Valle, Maria-Jose, "Increasing Inventories: The Role of Delivery Times," *mimeo University of Minnesota*, 2022.

Cass, David, "On the Wicksellian Point-Input, Point-Output Model of Capital Accumulation: A Modern View (or, Neoclassicism Slightly Vindicated)," *Journal of Political Economy*, 1973, *81* (1), 71–97.

Chor, Davin, "Unpacking sources of comparative advantage: A quantitative approach," *Journal of International Economics*, 2010, *82* (2), 152–167.

Costinot, Arnaud, "An elementary theory of comparative advantage," *Econometrica*, 2009, *77* (4), 1165–1192.

_ , "On the Origins of Comparative Advantage," *Journal of International Economics*, 2009, *77* (2), 255–264.

Deardorff, Alan V, "The possibility of factor price equalization, revisited," *Journal of International Economics*, 1994, *36* (1-2), 167–175.

_ , "Time and Trade: The Role of Time in Determining the Structure and Effects of International Trade, with an Application to Japan," *Japan's Economic Recovery*, 2003, pp. 63–76.

Dixit, Avinash and Victor Norman, *Theory of International Trade: A Dual, General Equilibrium Approach*, Cambridge University Press, 1980.

Djankov, Simeon, Caralee McLiesh, and Andrei Shleifer, "Private Credit in 129 Countries," *Journal of Financial Economics*, 2007, *84* (2), 299–329.

_ **, Caroline Freund, and Cong S Pham**, "Trading on Time," *The Review of Economics and Statistics*, 2010, *92* (1), 166–173.

Dorfman, Robert, "Waiting and the Period of Production," *The Quarterly Journal of Economics*, 1959, *73* (3), 351–372.

Dornbusch, Rudiger, Stanley Fischer, and Paul Samuelson, "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 1977, *67* (5), 823–839.

_ , _ , and _ , "Heckscher-Ohlin trade theory with a continuum of goods," *The Quarterly Journal of Economics*, 1980, *95* (2), 203–224.

Eppinger, Peter S. and Katja Neugebauer, "External financial dependence and firms' crisis performance across Europe," *Empirical Economics*, 2022, *62* (2), 887–904.

Ethier, Wilfred, "The theorems of international trade in time-phased economies," *Journal of International Economics*, 1979, *9* (2), 225–238.

Evans, Carolyn L and James Harrigan, "Distance, Time, and Specialization: Lean Retailing in General Equilibrium," *American Economic Review*, 2005, *95* (1), 292–313.

Feenstra, Robert C., Robert Inklaar, and Marcel P. Timmer, "The Next Generation of the Penn World Table," *American Economic Review*, 2015, *105* (10), 3150–3182.

Ferrari, Alessandro, "Inventories, Demand Shocks Propagation and Amplification in Supply Chains," *arXiv preprint arXiv:2205.03862*, 2022.

Findlay, Ronald, "An 'Austrian' Model of International Trade and Interest Rate Equalization," *Journal of Political Economy*, 1978, *86* (6), 989–1007.

Gaulier, Guillaume and Soledad Zignago, "BACI: International Trade Database at the Product-Level. The 1994-2007 Version," Working Paper 2010-23, CEPII 2010.

Harms, Philipp, Oliver Lorz, and Dieter Urban, "Offshoring along the Production Chain," *Canadian Journal of Economics*, 2012, *45* (1), 93–106.

Helpman, Elhanan and Paul Krugman, *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy*, MIT press, 1985.

Hicks, John R., *Value and Capital*, Oxford: Clarendon Press, 1939.

Hummels, David L and Georg Schaur, "Time as a Trade Barrier," *American Economic Review*, 2013, *103* (7), 2935–59.

Jacobson, Tor and Erik Von Schedvin, "Trade Credit and the Propagation of Corporate Failure: An Empirical Analysis," *Econometrica*, 2015, *83* (4), 1315–1371.

Jevons, William Stanley, *The Theory of Political Economy*, Macmillan & Co., 1871.

Kim, Se-Jik and Hyun Song Shin, "Theory of Supply Chains: A Working Capital Approach," *BIS Working Paper No. 1070*, 2023.

Klapper, Leora, Luc Laeven, and Raghuram Rajan, "Trade Credit Contracts," *The Review of Financial Studies*, 2011, *25* (3), 838–867.

Levchenko, Andrei A., "Institutional Quality and International Trade," *The Review of Economic Studies*, 2007, *74* (3), 791–819.

Lewin, Peter and Nicolás Cachanosky, "The average period of production: The history and rehabilitation of an idea," *Journal of the History of Economic Thought*, 2018, *40* (1), 81–98.

Liao, Steven, In Song Kim, Sayumi Miyano, and Hao Zhang, *concordance: Product Concordance* 2020. R package version 2.0.0.

Macaulay, Frederick R., *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856*, New York: National Bureau of Economic Research, 1938.

Manova, Kalina, "Credit Constraints, Heterogeneous Firms, and International Trade," *The Review of Economic Studies*, 2013, *80* (2), 711–744.

Marx, Karl, *Capital: A Critique of Political Economy, Volume I*, 1st ed., New York, NY: Cosimo, 1867.

Mas-Colell, Andreu, "Capital theory paradoxes: anything goes," in "Joan Robinson and modern economic theory," Springer, 1989, pp. 505–520.

Metzler, Lloyd, "The Rate of Interest and the Marginal Product of Capital," *Journal of Political Economy*, 1950, *58* (4), 289–306.

_ , "Wealth, Saving, and the Rate of Interest," *Journal of Political Economy*, 1951, *59* (2), 93–116.

Mundell, Robert A, "International Trade and Factor Mobility," *American Economic Review*, 1957, *47* (3), 321–335.

Nunn, Nathan, "Relationship-Specificity, Incomplete Contracts, and the Pattern of Trade," *Quarterly Journal of Economics*, 2007, *122* (2), 569–600.

_ **and Daniel Trefler**, "Domestic institutions as a source of comparative advantage," *Handbook of International Economics*, 2014, *4*, 263–315.

Petersen, Mitchell A. and Raghuram G. Rajan, "Trade Credit: Theories and Evidence," *The Review of Financial Studies*, 2015, *10* (3), 661–691.

Petith, Howard, "Vintage Capital, Joint Production and the Theory of International Trade," *International Economic Review*, 1972, pp. 148–159.

Pisch, Frank, "Managing Global Production: Theory and Evidence from Just-in-Time Supply Chains," *CEP Discussion Paper No 1689*, 2020.

Rajan, Raghuram G. and Luigi Zingales, "Financial Dependence and Growth," *The American Economic Review*, 1998, *88* (3), 559–586.

Ramos-Menchelli, Daniel and Bernardus Van Doornik, "The Spatial Consequences of Financial Frictions: Evidence From Brazil," *mimeo Harvard University*, 2022.

Ricardo, David, *On the Principles of Political Economy and Taxation*, London, UK: John Murray, 1817.

Romalis, John, "Factor Proportions and the Structure of Commodity Trade," *The American Economic Review*, 2004, *94* (1), 67–97.

Sraffa, Piero, *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory*, London: Cambridge University Press, 1960.

Steedman, Ian and John S Metcalfe, "Reswitching, primary inputs and the Heckscher-Ohlin-Samuelson theory of trade," *Journal of International Economics*, 1977, *7* (2), 201–208.

Swan, Trevor Winchester, "Economic growth and capital accumulation," *Economic Record*, 1956, *32* (2), 334–361.

Tyazhelnikov, Vladimir, "Production Clustering and Offshoring," *American Economic Journal: Microeconomics*, 2022, *14* (3), 700–732.

von Weizsäcker, Carl Christian, *Steady state capital theory*, Springer, 1971.

Wasserman, Janek, *The Marginal Revolutionaries*, Yale University Press, 2019.

Waugh, Michael E., "International Trade and Income Differences," *American Economic Review*, December 2010, *100* (5), 2093–2124.

Wicksell, Knut, *Lectures on Political Economy*, London: Routledge & Sons, Ltd., 1934.

World Bank, "Global Financial Development Database," 2023.

A Theoretical Appendix

A.1 Proof of Proposition 1

Differentiating equation (4) delivers

$$\frac{\partial \ln c(w,r)}{\partial r} = \frac{\int_0^T w \frac{\partial a_L(r,t)}{\partial r} e^{r(T-t)} dt}{c(w,r)} + \frac{\int_0^T (T-t)\, w a_L(r,t)\, e^{r(T-t)} dt}{c(w,r)},$$

but the first term is necessarily zero by the envelope theorem because the vector of unit labor requirements $a_{\mathbf{L}}(r) = \{a_L(r,t)\}_{t\in[0,T]}$ is set to minimize $c(w,r)$, while the second term corresponds exactly to the definition of the $\mathcal{APP}$ in equation (2).

A.2 Proof of Proposition 2

The result follows from straightforward differentiation of equation (9). In particular:

$$\begin{aligned}
\frac{\partial \ln w}{\partial r} &= -\int_0^1 \beta_v \left(\frac{\int_0^{T_v} \frac{\partial a_{Lv}(t,r)}{\partial r} e^{r(T_v-t)} dt}{c_v(w,r)/w} + \frac{\int_0^{T_v} (T_v-t)\, a_{Lv}(t,r)\, e^{r(T_v-t)} dt}{c_v(w,r)/w} \right) dv \\
&= -\int_0^1 \beta_v \mathcal{APP}_v dv,
\end{aligned}$$

where the second line follows from the envelope theorem – as in Proposition 1 – and from the definition of the $\mathcal{APP}$ in sector v.

A.3 Proof of Proposition 3

Starting from equation (24), take natural logs to obtain:

$$\Theta = \ln\left(\frac{p_v^F/p_v^H}{p_{v'}^F/p_{v'}^H}\right) = \ln \tilde{a}_{Lv}^F\left(r^F\right) - \ln \tilde{a}_{Lv}^H\left(r^H\right) - \left(\ln \tilde{a}_{Lv'}^F\left(r^F\right) - \ln \tilde{a}_{Lv'}^H\left(r^H\right)\right). \tag{A.1}$$

Given Proposition 1, if $r^F > r^H$, then Θ is positive if and only if $\mathcal{APP}_v^j > \mathcal{APP}_{v'}^j$. In the absence of $\mathcal{APP}$ reversals we can safely re-order sectors such that the index v associated with goods is such that $\mathcal{APP}_v^j$ is decreasing in v. Thus, $\mathcal{APP}_v^j > \mathcal{APP}_{v'}^j$ if and only if $v < v'$. We can then conclude that, whenever $r^F > r^H$, Θ in equation (A.1) is necessarily positive for all v below a given threshold v^*, and for $v > v^*$. In words, the lower interest rate country (Home) necessarily has comparative advantage in sectors with relatively long average production (or low v sectors).

A.4 Proof of Proposition 4

Consider first a (candidate) equilibrium with complete specialization and $r^H < r^F$. As shown in the main text, the intersection of the $A(v)$ and $B(v)$ schedules delivers a unique threshold good v^* as a function of the interest rates r^H and r^F because the $A(v)$ schedule is downward sloping, while the $B(v)$ schedule is upward sloping and satisfies $B(v^*) \to 0$ as $v^* \to 0$, and $B(v^*) \to \infty$ when $v^* \to 1$. Let us denote this intersection with $v^*\left(r^H, r^F\right)$. As pointed out in the main text, this same system together with the choice of numéraire also allows us to solve for w^H and w^F, as a function of r^H and r^F (see footnote 21). Because the dependence of the schedule $B(v^*)$ on r^H and r^F is generally ambiguous, it is however not possible to generally sign the impact of these interest rates on relative wages and the threshold v^* (more on this below).

Next consider the capital-market clearing conditions (28) and (29) for the (candidate) case $r^H < r^F$.

$$\int_0^{v^*} \frac{L_v^H}{L^H} \times \frac{K_v^H}{L_v^H} dv = \frac{w^H}{\phi^H - r^H} = \frac{K^{sH}}{L^H} \tag{A.2}$$

and

$$\int_{v^*}^{1} \frac{L_v^F}{L^F} \times \frac{K_v^F}{L_v^F} dv = \frac{w^F}{\phi^F - r^F} = \frac{K^{sF}}{L^F}. \tag{A.3}$$

These constitute two equations in two unknowns, since v^*, w^H and w^F are all functions of r^H and r^F, and L_v^j and K_v^j/L_v^j are also a function of parameters and factor prices. What can we say about this system?

Note first that the left-hand side terms of these equations are employment-weighted averages of capital intensities in the sectors produced in each country, under the maintained assumption that Home is the lower-interest rate country and thus produces in the sectors with long $\mathcal{APP}$s (or low index v sectors). In regular economies, these weighted averages will be higher in equation (A.2) than in equation (A.3) for two reasons. First, because Home features a lower interest rate, and thus operates production technologies at higher capital intensity. Second, because Home specializes in sectors that are relatively capital intensive (given the absence of time-versus-capital intensity ranking reversals in regular economies).

Despite these forces, as long as Home remains relatively capital abundant with free trade (i.e., $K^{sH}/L^H > K^{sF}/L^F$), it is possible for these two equations to deliver equilibrium interest rates r^H and r^F satisfying $r^H < r^F$. We have not been able to demonstrate the existence or uniqueness of such type of equilibrium in full generality. As discussed in the main text, this is for at least two reasons: first, because how v^* depends on r^H and r^F depends on subtle aspect of time-phased production (see the example in footnote 12 in the main text); and second, because capital supply is endogenous in our model, and in some cases, it may be backward bending with respect to interest rates (because capital supply depends on wages, and these are negatively related to interest rates). Nevertheless, it is straightforward to numerically construct and verify the uniqueness of equilibria for particular specifications of the model, as exemplified by our Wicksell-Swan Model and Proposition 5. This establishes the *possible* existence of the type of complete-specialization equilibrium described in part (a) of the Proposition.

We next prove that, in regular economies, we can rule out equilibria with factor price differences across countries in which the relatively capital scarce country specializes in sectors with long-$\mathcal{APP}$s. The reason for this is simple: if $K^{sH}/L^{H} > K^{sF}/L^{F}$ and Foreign were to specialize in long-$\mathcal{APP}$ sectors, then the ratio of the modified equilibrium conditions (A.2) and (A.3) would imply

$$\frac{\int_{v*}^{1} \frac{L_v^H}{L^H} \times \frac{K_v^H}{L_v^H} dv}{\int_{0}^{v*} \frac{L_v^F}{L^F} \times \frac{K_v^F}{L_v^F} dv} = \frac{K^{sH}/L^{H}}{K^{sF}/L^{F}},$$

which cannot possibly hold because the left-hand-side of this expression is lower than one (since, in the absence of time-versus-capital intensity ranking reversals, short-$\mathcal{APP}$ sectors feature lower capital intensity), while the right-hand side is higher than one by assumption. As long as Home remains relatively capital abundant, it cannot end up importing goods with long-$\mathcal{APP}$s. The only way for Foreign to be a net exporter of these long-$\mathcal{APP}$ goods is for it to be relatively capital abundant.

We finally consider equilibria with factor price equalization (or FPE). This corresponds to equilibria of an integrated equilibrium in which capital and labor can freely move across countries. The total endowments of capital and labor are given by $\bar{L} = L^{H} + L^{F}$, and $\bar{K} = \bar{w}L^{H}/\left(\phi^{H} - \bar{r}\right) + \bar{w}L^{F}/\left(\phi^{F} - \bar{r}\right)$, where $\bar{w}$ and $\bar{r}$ are the equilibrium factor prices in the integrated equilibrium. These can easily be computed from the closed-economy of our model from equations (9) and (15). For this to be an equilibrium, the Home and Foreign countries need to allocate capital and labor across sectors in a way that clears those local markets at the integrated factor prices $\bar{w}$ and $\bar{r}$. Define the allocation of capital of Home and Foreign in sector v by K_v^H and K_v^F and the analogous allocations for labor as L_v^H and L_v^F.

For the Home capital market to clear, we need

$$\int_0^1 K_v^H dv = \int_0^1 \int_0^{T_v} \bar{w}\bar{\ell}_v^H(t) \frac{e^{\bar{r}(T_v - t)} - 1}{\bar{r}} dt = \frac{\bar{w}L^H}{\phi^H - \bar{r}} ,$$

or

$$\int_0^1 \frac{L_v^H}{L^H} \times \frac{K_v^H}{L_v^H} dv = \frac{\bar{w}}{\phi^H - \bar{r}} , \tag{A.4}$$

while for Foreign, we need

$$\int_0^1 \frac{L_v^F}{L^F} \times \frac{K_v^F}{L_v^F} dv = \frac{\bar{w}}{\phi^F - \bar{r}} . \tag{A.5}$$

An equilibrium with FPE needs to satisfy the following additional constraints:

$$\int_0^1 L_v^H dv = L^H \tag{A.6}$$

$$\int_0^1 L_v^F dv = L^F \tag{A.7}$$

$$\frac{L_v^H + L_v^F}{L^H + L^F} = \frac{\Lambda_v(\bar{r})}{\int_0^1 \Lambda_v(\bar{r})\, dv}. \tag{A.8}$$

It is well-known that there are a continuum of possible allocations L_v^H and L_v^F satisfying these conditions. It is also well-known, and easily verified, that whenever $\phi^H = \phi^F$, an equilibrium with FPE will necessarily exist. To see this, it suffices to set

$$L_v^H = \frac{\Lambda_v(\bar{r})}{\int_0^1 \Lambda_v(\bar{r})\, dv} L^H; \quad L_v^F = \frac{\Lambda_v(\bar{r})}{\int_0^1 \Lambda_v(\bar{r})\, dv} L^F,$$

and verify that all the above conditions (A.6), (A.7) and (A.8) for an FPE are satisfied, and that indeed, equations (A.4) and (A.5) are consistent with capital market clearing for the integrated economy, i.e.,

$$\int_0^1 K_v^H dv + \int_0^1 K_v^F dv = \frac{\bar{w}}{\phi - \bar{r}} \left(L^H + L^F\right).$$

In sum, for $\phi^H = \phi^F$, an equilibrium with FPE exists. Note also that when $\phi^H = \phi^F$, in an equilibrium with FPE, the aggregate capital labor ratios of the two countries will be identical. It is then straightforward to marginally perturb the labor and capital market vectors L_v^j, K_v^j for $j = H, F$, to sustain an FPE equilibrium even when $K^{sH}/L^H \neq K^{sF}/L^F$. In words, an equilibrium with FPE can be constructed when aggregate capital-labor ratios are sufficiently similar in the two countries.

We finally show that, when aggregate capital-labor ratios are sufficiently similar in the two countries, an FPE equilibrium is the *only* type of possible equilibria. Suppose then that $K^{sH}/L^H = K^{sF}/L^F(1+\varepsilon)$ for sufficiently small ε, but $r^H < r^F$ (the case $r^H > r^F$ can be analyzed analogously). In such a case, Home would specialize in sectors with low index v (or long-$\mathcal{APP}$ sectors). But then, the average capital intensity of Home's production sectors would be higher than Foreign's, which would violate the capital market clearing condition for sufficiently small ε, or

$$\frac{\int_0^{v*} \frac{L_v^H}{L^H} \times \frac{K_v^H}{L_v^H} dv}{\int_{v*}^1 \frac{L_v^F}{L^F} \times \frac{K_v^F}{L_v^F} dv} > 1 + \varepsilon = \frac{K^{sH}/L^H}{K^{sF}/L^F}.$$

Thus, for K^{sH}/L^H sufficiently close to K^{sF}/L^F, the only candidate equilibrium is one with FPE. This confirms the last statement of Proposition 4.

It is important to note that we cannot rule out the existence of non-FPE equilibria in the neighborhood of $\phi^H = \phi^F$ because the endogenous nature of capital supply might give rise to equilibria in which a country specializes in capital-intensive sectors, which bids up interest rates and leads to a relatively higher capital-labor ratio for this country, which in turn permits a clearing of factor markets even when $r^H < r^F$. This is the reason for why the results in Proposition 4 are stated in terms of asymmetries in equilibrium aggregate capital-labor ratios (rather than in terms of primitive parameters, such as ϕ^H or ϕ^F).

The above caveat is of course irrelevant in the fixed-capital-supply variant of our model (see Online Appendix C.3), in which relative aggregate capital-labor ratios are instead pinned down by primitives $(K^H/L^H, K^F/L^F)$, and the corresponding analogue of Proposition 4 can be stated directly in terms of these exogenous endowments. While multiplicity may still arise within the factor-price-equalization set due to the usual indeterminacy of specialization patterns, any equilibrium featuring factor-price differences must assign the country with the higher aggregate capital–labor ratio to the relatively long-$\mathcal{APP}$ (working-capital-intensive) goods. See Online Appendix C.3 for the equilibrium characterization and an Edgeworth-box illustration.

A.5 Proof of Proposition 5

Consider first a (candidate) equilibrium with complete specialization and $r^F > r^H$. As shown in the main text, the intersection of the $A(v)$ and $B(v)$ schedules delivers a unique threshold good v^* as a function of the interest rates r^H and r^F. Furthermore, because the $B(v)$ schedule in equation (30) is independent of these interest rates, while the schedule $A(v)$ is given by $A(v) = \left(r^F/r^H\right)^{\zeta_v}$, we can express v^* as a function of the interest-rate ratio r^H/r^F, with $v^*\left(r^H/r^F\right)$ decreasing in r^H and increasing in r^F. Notice also that, when $r^H/r^F \to 0$, then $v^* \to 1$, and when $r^H/r^F \to +\infty$, then $v^* \to 0$. More specifically, we have

$$\frac{r^H}{r^F} = \left(\frac{\int_0^{v^*} \beta_v e^{-\zeta_v} dv}{\int_{v^*}^1 \beta_v e^{-\zeta_v} dv} \frac{L^F}{L^H}\right)^{-1/\zeta_{v^*}} = g\left(v^*\right), \tag{A.9}$$

where the right-hand-side is decreasing in v^* because $r^H < r^F$ in this candidate equilibrium – so the term in parenthesis must be higher than one – and because ζ_{v^*} is decreasing in v^*.[43]

Next, notice that we can combine the capital-market clearing conditions (31) and (32) to

[43]More formally, the right hand side takes the form $g\left(v^*\right) = \left(h\left(v^*\right)\right)^{-1/n\left(v^*\right)}$, so

$$g'\left(v^*\right) = -\left[n\left(v^*\right)h'\left(v^*\right) + h\left(v^*\right)\left(\ln\left(h\left(v^*\right)\right)\right)n'\left(v^*\right)\right]\frac{g\left(v^*\right)}{h\left(v^*\right)\left(n(v^*)\right)^2} < 0,$$

where the sign follows from $n\left(v^*\right) > 0$, $h'\left(v^*\right) > 0$, $h\left(v^*\right) > 1$ and $n'\left(v^*\right) < 0$.

obtain

$$\frac{r^H}{r^F} = \frac{\phi^H}{\phi^F} \times \frac{\int_0^{v^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_0^{v^*} \beta_v dv}{\int_{v^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_{v^*}^1 \beta_v dv} = f\left(v^*\right). \tag{A.10}$$

This equation delivers a second equation $f(v^*)$ linking the ratio r^H/r^F to v^*. Notice first that the second term in the right-hand-side of this equation is necessarily higher than one because ζ_v is decreasing in v: in more plain words, low v sectors are both more time- and capital-intensive, and thus the average capital intensity of production is higher at Home than in Foreign. An immediate implication of this fact is that, if ϕ^H/ϕ^F is close enough to 1, and equilibrium with $r^H < r^F$ cannot exist because equation (A.10) would imply $r^H/r^F > 1$ for $\phi^H/\phi^F \simeq 1$.

Next, it may seem intuitive that the right-hand-side of equation (A.10) would be increasing in v^*, since a larger range of Home-produced goods should bid up the relative demand for Home capital and its relative interest rate. Indeed, this function is increasing for many parameterizations of the functions β_v and ζ_v. In those circumstances, the curves defined by (A.9) and (A.10) must intersect once, and thus the equilibrium exists and is unique, since once r^H/r^F and v^* are uniquely pinned down, it is straightforward to solve for the unique level of all factor prices.[44] For instance, when $\beta_v = 1$ for all v and $\zeta_v = 1 - v$, as in the numerical simulations at the end of Section 4.3, the schedule is indeed increasing. More specifically,

$$f\left(v^*\right) = \frac{\phi^H}{\phi^F} \frac{\int_0^{v^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_0^{v^*} \beta_v dv}{\int_{v^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_{v^*}^1 \beta_v dv} = \frac{\phi^H}{\phi^F} \left(1 - v^*\right) \frac{v^* + e^{-1} - e^{v^*-1}}{v^* \left(e^{v^*-1} - v^*\right)}$$

and it is straightforward (though tedious) to show that $f'\left(v^*\right) > 0$ for $v^* \in [0, 1]$.

Nevertheless, for other choices of β_v and ζ_v, the right-hand-side of equation (A.10) may *not* be increasing in v^*. The reason for this is that a higher v^* not only raises the relative demand for capital at Home, but it also raises its relative supply because remember that savings are proportional to labor income. The endogenous nature of capital supply thus complicates the proof of existence and uniqueness relative to an analogous Heckscher-Ohlin framework.

In contrast, in the fixed-capital-supply variant in Online Appendix C.3, this additional supply channel is absent because aggregate capital endowments do not respond to wages or to specialization. In that case, the analogue of the schedule $f(v^*)$ reflects only relative capital *demand* and is monotone under the same regularity conditions (e.g., declining capital intensity in r), which greatly simplifies uniqueness arguments (up to the standard indeterminacy of specialization patterns inside the FPE set). We discuss this in more detail in Online Appendix C.3.

Despite these complications, notice that, when evaluated at $v^* = 0$, the right-hand-side of

[44] Specifically, r^H and r^F can be solved from (31) and (32), respectively. The ratio w^H/w^F can be obtained from $w^H/w^F = A\left(v^*\right) = \left(r^F/r^H\right)^{\zeta_v *}$, and the level of one of these wage rates can be backed out from the equation for the ideal price index, which is the numéraire (see footnote 21 in the main text).

equation (A.10) equals

$$\lim_{v^* \to 0} f(v^*) = \frac{\phi^H}{\phi^F} \frac{1 - e^{-\zeta_0}}{\int_0^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv} \geqslant 0,$$

which necessarily takes a non-negative bounded value. Similarly,

$$\lim_{v^* \to 1} f(v^*) = \frac{\phi^H}{\phi^F} \frac{\int_0^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{1 - e^{-\zeta_1}} > 0,$$

which is also necessarily strictly positive. Because the schedule $g(v^*)$ associated with equation (A.9) runs from $+\infty$ ($v^* = 0$) to 0 ($v^* = 1$), there must be at least one intersection of the $f(v^*)$ and $g(v^*)$ schedules. When that intersection occurs for a value of $r^H/r^F < 1$, this constitutes an equilibrium with factor price differences across countries. Note again that we cannot generally rule out that more than one intersection occurs due to the endogeneity of capital supplies (though see Online Appendix C.3).

What happens then when equations (A.9) and (A.10) deliver an intersection for which $r^H > r^F$. This is of course not an equilibrium because the equations that delivered this system imposed that Home specialized in the low v sectors. And given Proposition 3, $r^H > r^F$ is only consistent with Foreign specializing in the low v sectors. Following the same steps as in the construction of the candidate equilibrium with $r^H < r^F$, if Foreign specializes in low v sectors, this would result in an equation analogous to (A.10) but given by

$$\frac{r^F}{r^H} = \frac{\phi^F}{\phi^H} \times \frac{\int_0^{v'^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_0^{v'^*} \beta_v dv}{\int_{v'^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_{v'^*}^1 \beta_v dv},$$

for some $v'^* \in (0, 1)$. But note that, whenever $\phi^H < \phi^F$, the right-hand-side of this equation is a product of two terms that are higher than one, and thus this equation would deliver $r^F > r^H$, contradicting the starting assumption of this alternative candidate equilibrium. In sum, whenever $\phi^H < \phi^F$, we cannot have an equilibrium with $r^H > r^F$.

It remains to study the possibility of an equilibrium with factor price equalization. We can construct such an equilibrium in a manner completely analogous to that in the proof of Proposition 4. Consider then an integrated equilibrium in which capital and labor could freely move across countries. The total endowments of capital and labor are given by $\bar{L} = L^H + L^F$, and $\bar{K} = \bar{w} L^H / \left(\phi^H - \bar{r}\right) + \bar{w} L^F / \left(\phi^F - \bar{r}\right)$, where $\bar{w}$ and $\bar{r}$ are the equilibrium factor prices in the integrated equilibrium. These can easily be computed from the closed-economy of our model from equation (9), i.e.,

$$\ln \bar{w} = \int_0^1 \beta_v \left(\zeta_v \ln \left(\zeta_v / \bar{r}\right) - \zeta_v - \ln \beta_v\right) dv,$$

and from equilibrium in the world capital market, or

$$\frac{\int_0^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^1 \beta_v e^{-\zeta_v} dv} = \frac{\bar{r}}{\phi^H - \bar{r}} \frac{L^H}{\bar{L}} + \frac{\bar{r}}{\phi^F - \bar{r}} \frac{L^F}{\bar{L}}.$$

For this to be an equilibrium, the Home and Foreign countries need to allocate capital and labor across sectors in a way that clears those local markets at the integrated factor prices $\bar{w}$ and $\bar{r}$. For the Home capital market to clear, we need

$$\int_0^1 K_v^H dv = \int_0^1 L_v^H \frac{1 - e^{-\zeta_v}}{e^{-\zeta_v}} \frac{\bar{w}}{\bar{r}} dv = \frac{\bar{w} L^H}{\phi^H - \bar{r}}.$$

or

$$\int_0^1 \frac{L_v^H}{L^H} \frac{1 - e^{-\zeta_v}}{e^{-\zeta_v}} dv = \frac{\bar{r}}{\phi^H - \bar{r}},$$

while for Foreign, we need

$$\int_0^1 \frac{L_v^F}{L^F} \frac{1 - e^{-\zeta_v}}{e^{-\zeta_v}} dv = \frac{\bar{r}}{\phi^F - \bar{r}}.$$

An equilibrium with FPE needs to satisfy the following additional constraints:

$$\begin{aligned} \int_0^1 \frac{L_v^H}{L^H} dv &= 1 \\ \int_0^1 \frac{L_v^F}{L^H} dv &= 1 \\ \frac{L_v^H + L_v^F}{L^H + L^F} &= \frac{\beta_v e^{-\zeta_v}}{\int_0^1 \beta_v e^{-\zeta_v} dv}. \end{aligned}$$

As is well-known, there are a continuum of possible allocations L_v^H and L_v^F satisfying these conditions. Furthermore, it is easily verified that whenever $\phi^H = \phi^F$, an equilibrium with FPE will necessarily exist. Just set

$$L_v^H = \frac{\beta_v e^{-\zeta_v}}{\int_0^1 \beta_v e^{-\zeta_v} dv} L^H; \quad L_v^F = \frac{\beta_v e^{-\zeta_v}}{\int_0^1 \beta_v e^{-\zeta_v} dv} L^F$$

and verify that all the conditions for an FPE are satisfied.

Equilibria with FPE exist for asymmetric patience rates and levels of financial development, and the maximal asymmetry in ϕ^H and ϕ^F consistent with FPE can be computed by studying the case in which Home specializes in the most capital intensive sectors ($v < v^*$) while Foreign specializes in the least capital intensive goods, and still factor markets clear at $\bar{w}$ and $\bar{r}$. This corresponds to the so-called 'lens condition' in Deardorff (1994). This delivers

$$
\begin{aligned}
L_v^H &= \frac{\beta_v e^{-\zeta_v}}{\int_0^{v^*} \beta_v e^{-\zeta_v} dv} L^H \text{ for } v \leqslant v^*;\ L_v^H = 0 \text{ otherwise;} \\
L_v^F &= \frac{\beta_v e^{-\zeta_v}}{\int_{v^*}^1 \beta_v e^{-\zeta_v} dv} L^F \text{ for } v > v^*;\ L_v^F = 0 \text{ otherwise,}
\end{aligned}
$$

with the threshold v^* given by

$$
\frac{L^H}{L^H + L^F} = \frac{\int_0^{v^*} \beta_v e^{-\zeta_v} dv}{\int_0^1 \beta_v e^{-\zeta_v} dv}. \tag{A.11}
$$

The associated capital-labor ratios used at Home and in Foreign in this equilibrium are

$$
\frac{K^{dH}}{L^H} = \frac{\bar{w}}{\bar{r}} \frac{\int_0^{v^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{v^*} \beta_v e^{-\zeta_v} dv}
$$

and,

$$
\frac{K^{dH}}{L^H} = \frac{\bar{w}}{\bar{r}} \frac{\int_{v^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_{v^*}^1 \beta_v e^{-\zeta_v} dv}
$$

respectively.

Whenever $\phi^F > \phi^H$, FPE is then attained whenever the Home and Foreign capital-labor ratio fall within this FPE lens, or

$$
\frac{\int_0^{v^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{v^*} \beta_v e^{-\zeta_v} dv} \geqslant \frac{\bar{r}}{\phi^H - \bar{r}}
$$

and

$$
\frac{\int_{v^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_{v^*}^1 \beta_v e^{-\zeta_v} dv} \leqslant \frac{\bar{r}}{\phi^F - \bar{r}},
$$

where v^* is given by (A.11).

Let us finally draw the implications of our results above. Define the 'parameter space' as all possible combinations of parameters of the model $\left(\phi^H, \phi^F, L^H, L^F, \boldsymbol{\zeta}, \boldsymbol{\beta}\right)$. For each combination of these parameters, we can then define the pair $(\bar{r}, \bar{v}^*)$ satisfying

$$
\frac{\int_0^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^1 \beta_v e^{-\zeta_v} dv} = \frac{\bar{r}}{\phi^H - \bar{r}} \frac{L^H}{\bar{L}} + \frac{\bar{r}}{\phi^F - \bar{r}} \frac{L^F}{\bar{L}}. \tag{A.12}
$$

and

$$\frac{L^H}{L^H + L^F} = \frac{\int_0^{\bar{v}^*} \beta_v e^{-\zeta_v} dv}{\int_0^1 \beta_v e^{-\zeta_v} dv}. \tag{A.13}$$

This gives the interest rate associated with an integrated economy equilibrium, and the threshold good $\bar{v}^*$ associated with the boundary of the FPE set for the values of L^H and L^F in that particular combination of parameters.

Given these values $\bar{r}$ and $\bar{v}^*$, there can only be two mutually exclusive situations. First, if

$$\frac{\int_0^{\bar{v}^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{\bar{v}^*} \beta_v e^{-\zeta_v} dv} \geqslant \frac{\bar{r}}{\phi^H - \bar{r}}, \tag{A.14}$$

then an equilibrium with FPE necessarily exists, with an integrated equilibrium interest rate given by $\bar{r}$. In particular, we can show that

$$\frac{\int_{\bar{v}^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_{\bar{v}^*}^1 \beta_v e^{-\zeta_v} dv} \leqslant \frac{\bar{r}}{\phi^F - \bar{r}},$$

whenever (A.13), (A.12) and (A.14) hold.

Conversely, when

$$\frac{\int_0^{\bar{v}^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{\bar{v}^*} \beta_v e^{-\zeta_v} dv} < \frac{\bar{r}}{\phi^H - \bar{r}}, \tag{A.15}$$

we can show that an equilibrium without FPE necessarily exists. To see this, note that equation (A.15), together with (A.12) and (A.13), implies

$$f\left(\bar{v}^*\right) = \frac{\phi^H}{\phi^F} \times \frac{\int_0^{\bar{v}^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_0^{\bar{v}^*} \beta_v dv}{\int_{\bar{v}^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv / \int_{\bar{v}^*}^1 \beta_v dv} < 1.$$

Now from the definition of $\bar{v}^*$ in (A.13), we also know $g\left(\bar{v}^*\right) = 1$. Hence, at $\bar{v}^*$, we necessarily have $f\left(\bar{v}^*\right) - g\left(\bar{v}^*\right) < 0$. Conversely, as $v^* \to 1$, we know that $g\left(v^*\right) \to 0$, while $f(1)$ is a bounded positive number. Hence, $\lim_{v^* \to 1}\left(f\left(v^*\right) - g\left(\bar{v}^*\right)\right) > 0$. By the continuity of the functions $f\left(v^*\right)$ and $g\left(\bar{v}^*\right)$, there must then be a $v^* > \bar{v}^*$, such that the schedules $f\left(v^*\right)$ and $g\left(v^*\right)$ intersect, and furthermore, because the function $g\left(v^*\right)$ is decreasing, at that $v^* > \bar{v}^*$ we must necessarily have $r^H/r^F = g\left(v^*\right) < 1$. In sum, whenever condition (A.15) holds, an equilibrium without FPE exists. Because conditions (A.14) and (A.15) jointly cover the entire parameter space (they are converse inequalities), we can thus state in Proposition 5 that an equilibrium necessarily exists.

B Empirical Appendix

B.1 Data Construction

This section describes the construction of the variables for Section 5. Trade data comes from the CEPII-BACI database (Gaulier and Zignago, 2010). Specifically we use the 1996 vintage of six-digit HS codes covering 1996 through 2019. For details on the construction of $\mathcal{APP}$ see Antràs and Tubdenov (2025): we use the measures for 2012 NAICS industries derived from US firm-level data, as stated in the main text. We use concordance tables provided by Liao et al. (2020) to map trade flows at the six-digit HS level to six-digit NAICS industries. We successfully match all HS96 codes to NAICS industries. But of those NAICS industries, we only have a measure of the time intensity of production for industries accounting for 67.4% of trade volumes.

We use the same measure of financial development, $FD_{j,t}$, as Bilir et al. (2019): Private credit by deposit money banks to GDP(%). This is provided by the World Bank Global Financial Development Database, World Bank (2023).

Labor and capital controls are derived from the Penn World Tables (PWT), Feenstra et al. (2015), and NBER-CES database, Becker et al. (2021). We construct endowments from the PWT as

$$\frac{H_{i,t}}{L_{i,t}} = \log \mathrm{hc}_{i,t} \text{ and } \frac{K_{i,t}}{L_{i,t}} = \log \frac{\mathrm{cn}_{i,t}}{\mathrm{emp}_{i,t}} ,$$

where i indexes the country and t the year; hc is the human capital index (based on years of schooling and returns to education), emp is the number of persons engaged, and cn is the capital stock at current PPPs. From the NBER-CES database we construct skill and capital intensity as

$$\frac{H_v}{L_v} = \log \frac{EMP_v - PRODE_v}{EMP_v} \text{ and } \frac{K_v}{L_v} = \log \frac{CAP_v}{EMP_v} ,$$

where v indexes the industry; EMP is total employment, PRODE is production workers, and CAP is total real capital stock. Before applying logs, we take the median value of skill and capital intensity across years as the time-invariant sectoral measure.

To construct external capital dependence (EFD) we use Compustat data for North American firms. EFD is defined as the share of investment that is not financed with internal cash flows:

$$EFD_v = \frac{\text{Investment} - \text{Cash Flows}}{\text{Investment}}.$$

Investment is defined to be simply capital expenditure. Cash flows are defined following Eppinger and Neugebauer (2022) as either "the sum of income before extraordinary items (IBC), depreciation and amortization (DPC), deferred taxes (TXDC), equity in net loss/earnings

(ESUBC), sale of property, plant and equipment and investments (SPPIV), and other funds from operations (FOPO)" for SCF code 7, or "sum of total funds from operations (FOPT) plus increases in accounts payable (AP_t-AP_{t-1}), decreases in inventories (INVT_{t-1}-INVT_t), and decreases in receivables (RECT_{t-1}-RECT_t)" for SCF code 1. We calculate this measure for each firm-year, then take the median value across firms and years (1990 to 2019) to get a measure of external financial dependence by NAICS industry. Industries with fewer than 50 firm-year observations are dropped.

Our main analysis considers only observations for which the full set of interactions is available, and hence we drop industries where either data from the NBER-CES database or Compustat is unavailable. This requires we drop sectors representing 16.8% of trade volumes in the remaining sample. In Table B.1, we present a specification that adds back these sectors but omits these additional interactions.

B.2 Additional Empirical Results

In Table B.1 we examine the robustness of the main results presented in Table 1. The theory suggests that countries with lower capital costs would exhibit a comparative advantage in time-intensive sectors. However, it may be that either capital costs are correlated with other country-level characteristics or time intensity is correlated with other industry-level characteristics that are the true determinants of this comparative advantage. In column (1) we consider the full set of interactions between industry and country characteristics. The fact that many of these results are significant is not surprising, the country-level variables all exhibit some correlation with one another, as do the industry-level variables. Regardless, the direction and magnitude of the key interaction persists.

Another concern is simultaneity through the endogeneity of financial development. It might be the case that an increase in time-intensive exports frees up capital, which results in higher measured financial development. We can imagine similar stories for say capital accumulation or sectors with high external financial dependence. To break this simultaneity, we use lagged country-level characteristics in column (2).

Note that the sector-level characteristics are largely constructed from US data. Therefore, they may not accurately reflect the production technology available to other countries. So we next consider sample restrictions to limit our analysis to data where the sector-level characteristics are potentially more representative. Namely, in columns (3)-(6) we split the sample to above and below either the median export size or median GDP per capita. The main interaction seems to largely be driven by country-sectors with high export values and countries with high GDP per capita.

Next, in column (7), we consider a sample where we drop the sectors with $\mathcal{APP}$s in the

Table B.1: Comparative Advantage

	log Exports$_{v,j,t}$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\mathcal{APP}_v \times FD_{j,t}$	0.029***		0.089***	0.011	0.062***	0.012	0.044***	0.036***
	(0.008)		(0.017)	(0.010)	(0.009)	(0.010)	(0.008)	(0.006)
$K_v/L_v \times K_{j,t}/L_{j,t}$	0.134***		0.296***	-0.308***	-0.039	0.038	0.065**	
	(0.046)		(0.073)	(0.039)	(0.062)	(0.045)	(0.028)	
$H_v/L_v \times H_{j,t}/L_{j,t}$	-0.092***		0.245***	0.0003	0.054**	-0.064***	0.043***	
	(0.016)		(0.030)	(0.015)	(0.021)	(0.014)	(0.011)	
$EFD_v \times FD_{j,t}$	0.007		0.035***	0.008*	0.013**	0.004	0.012***	0.014***
	(0.005)		(0.012)	(0.004)	(0.006)	(0.005)	(0.004)	(0.004)
$\mathcal{APP}_v \times K_{j,t}/L_{j,t}$	-0.205***							
	(0.040)							
$\mathcal{APP}_v \times H_{j,t}/L_{j,t}$	0.034**							
	(0.017)							
$K_v/L_v \times FD_{j,t}$	-0.050***							
	(0.018)							
$K_v/L_v \times H_{j,t}/L_{j,t}$	-0.095***							
	(0.028)							
$EFD_v \times H_{j,t}/L_{j,t}$	0.010							
	(0.011)							
$H_v/L_v \times FD_{j,t}$	0.053***							
	(0.010)							
$H_v/L_v \times K_{j,t}/L_{j,t}$	0.253***							
	(0.035)							
$EFD_v \times K_{j,t}/L_{j,t}$	0.049							
	(0.033)							
$\mathcal{APP}_v \times FD_{j,t-1}$		0.028***						
		(0.006)						
$K_v/L_v \times K_{j,t-1}/L_{j,t-1}$		0.046*						
		(0.027)						
$H_v/L_v \times H_{j,t-1}/L_{j,t-1}$		0.021**						
		(0.011)						
$EFD_v \times FD_{j,t-1}$		0.016***						
		(0.004)						
Sample			Exports>Median	Exports<Median	GDPpc>Median	GDPpc<Median	APP Winsorized	
Observations	472,023	451,414	236,012	236,012	236,108	236,072	423,490	581,458
R^2	0.79	0.79	0.68	0.48	0.81	0.67	0.80	0.77
Country-Year fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
Industry-Year fixed effects	✓	✓	✓	✓	✓	✓	✓	✓

Note: ***: 0.01, **: 0.05, *: 0.1. Standard errors clustered at the country-industry level. Standardized beta coefficients are reported. The dependent variable, $\log \text{Exports}_{v,j,t}$, has a standard deviation of 4.00 in the sample.

bottom 5th or top 95th percentiles. Again, the main results persist.

Finally, note that the NBER CES database, which provides measures of capital and skill intensity, only covers manufacturing sectors. However, our measure of $\mathcal{APP}$ is available for non-manufacturing sectors (e.g. agriculture and resource sectors). In column (8) we run a specification that uses all sectors for which $\mathcal{APP}$ is available, but as a result requires us to drop the interactions which are no longer available.

C Online Appendix

C.1 Discrete Time

In this Appendix, we develop our model in discrete time. We have two objectives in mind. First, we want to show that our key results naturally generalize to this case, and that there is a new result that can be easily illustrated in that case (and not in our baseline model with continuous time). Second, we also show that the continuous-time formulation simplifies the algebra in one important way, which ultimately leads us to adopt it in the main text.

We will not develop the whole model in discrete time. We will simply illustrate how some key concepts generalize to a discrete-time setting, and will also study comparative advantage by analyzing relative cost differences under autarky, rather than developing the full general equilibrium in either its closed-economy or open-economy versions.

We now consider a model in which production takes place over a number of different periods $t = 0, 1, 2, ..., T$, where $t \in \mathbb{Z}^{0+}$. The mapping between inputs and output continues to be governed by the production function in (1) in the main text, or

$$y(\boldsymbol{L}) = ZF(\boldsymbol{L}), \tag{C.1}$$

except that L is now a *finite*-dimensional vector. The total cost of production – inclusive of cumulated interest costs – associated with this time-phased production process is given by

$$C(\boldsymbol{L}) = \sum_{t=0}^{T} w\ell_t (1+r)^{T-t},$$

where the last term in the sum captures the fact that wage payments in period t accumulate interest for a period $T - t$ before the firm collects revenue at the end of the process.

Given a path of labor used along the production process, we define:

Definition C.1. The average period of production or $\mathcal{APP}$ is given by:

$$\mathcal{APP}(\boldsymbol{L}) = \sum_{t=0}^{T} (T-t) \frac{w\ell_t (1+r)^{T-t}}{C(\boldsymbol{L})}. \tag{C.2}$$

This expression again connects with the one in Hicks (1939) and appropriately discounts labor expenditures at different points in time. The path of labor expenditures is in turn determined by a standard cost minimization problem

$$\begin{array}{ll} \min\limits_{\boldsymbol{L}} & \sum\limits_{t=0}^{T} w\ell\left(t\right)\left(1+r\right)^{T-t} \\ \text{s.t.} & \qquad y\left(\boldsymbol{L}\right) \geqslant 1 \end{array},$$

which delivers a vector of unit labor requirements $a_{\mathbf{L}}\left(r\right)=\left\{a_{L0}\left(r\right),a_{L1}\left(r\right),...,a_{LT}\left(r\right)\right\}$, which again only depend on the interest rate (as long as we focus on a stationary equilibrium with time-invariant wages). From this problem we also obtain a unit cost function

$$c\left(w,r\right)=\sum_{t=0}^{T} wa_{Lt}\left(r\right)\left(1+r\right)^{T-t},$$

as a function of both the interest rate and the wage rate.

Differentiating this cost function and invoking the envelope theorem, we can easily verify that

$$\frac{\partial \ln c\left(w,r\right)}{\partial r}=\frac{\mathcal{APP}}{1+r}.$$

In words, the percentage response of production costs to changes in the interest rate is proportional to the average period of production (or $\mathcal{APP}$), as defined in equation (C.2). This result is analogous to the result in Proposition 1, except that we now have a term $1+r$ in the denominator. This result is analogous to definition of (Macaulay's) duration in finance, which in its discrete time formulation also includes such a term in the denominator, an object that is often referred to as *modified duration*.

Comparative Advantage In a free trade equilibrium, the minimum price at which country j can produce good v can be expressed as

$$p_v^j=\tilde{a}_{Lv}^j\left(r^j\right)w^j,$$

where $\tilde{a}_L^j\left(r^j\right)$ is given by

$$\tilde{a}_L^j\left(r^j\right)=\sum_{t=0}^{T} a_{Lt}\left(r\right)\left(1+r\right)^{T-t}.$$

Given our result above, we now have that $\tilde{a}_L^j\left(r^j\right)$ satisfies

$$\frac{\partial \ln \tilde{a}_L^j\left(r^j\right)}{\partial r^j}=\frac{\mathcal{APP}_v^j}{1+r^j}.$$

Although this differs from equation (23) in the main text, it continues to be the case that $\tilde{a}_{Lv}^j\left(r^j\right)$ is log-supermodular in r^j and $\mathcal{APP}_v^j$, and thus we can continue to establish that

Proposition C.1. As long as there are no $\mathcal{APP}$ reversals, the country with the lower interest rate necessarily has comparative advantage in sectors with relatively long average production periods ($\mathcal{APP}$s).

The 'Failure' of Shephard's Lemma Up to this point, we have just established that some of the key properties of our continuous-time model are preserved when working in discrete time. We prefer the neater algebra associated with our continuous-time model, and this is why we adopt it in the main text, but, as pointed out by an anonymous referee, the continuous-time formulation obscures an important result related to the 'failure' of Shephard's lemma and the existence of time-versus-capital intensity ranking reversals in our framework.

To illustrate this, we compute capital demand by simply invoking the zero profit condition, which delivers

$$K^d = \frac{C(\boldsymbol{L}) - \sum_{t=0}^{T} w\ell_t}{r} = \frac{\sum_{t=0}^{T} w\ell_t \left[(1+r)^{T-t} - 1\right]}{r}. \tag{C.3}$$

We compare this to the capital demand that would be implied by an application of Shephard's lemma, or

$$\hat{K}^d = \frac{\partial C(\boldsymbol{L})}{\partial r} = \frac{\mathcal{APP}}{1+r} C(\boldsymbol{L}) = \sum_{t=0}^{T} (T-t) \frac{w\ell_t (1+r)^{T-t}}{1+r}. \tag{C.4}$$

We can write this as

$$\hat{K}^d = \sum_{t=0}^{T} w\ell_t \frac{(1+r)^{T-t} - 1}{r} \Upsilon$$

where

$$\Upsilon(t) = (T-t)(1+r)^{T-t-1} \frac{r}{(1+r)^{T-t} - 1}.$$

It is not totally obvious, but factorizing $(1+r)^{T-t}$, one can establish that $\Upsilon \geqslant 1$ whenever $T-t \geqslant 2$ and $r > 0$. Indeed, define $n = T-t$ and $x = 1+r$, then

$$\Upsilon(t) = nx^{n-1} \frac{x-1}{x^n - 1} = nx^{n-1} \frac{x-1}{(x-1)\sum_{m=0}^{n-1} x^k} = \frac{nx^{n-1}}{\sum_{m=0}^{n-1} x^k}.$$

For every $x \geqslant 1$, $x^k \leqslant x^{n-1}$ with equality only when $n = 1$ or $x = 1$. Therefore $\Upsilon \geqslant 1$, with equality only if $r = 0$ or $T - t = 1$.

The first result was established in our baseline model with continuous time: Shephard's lemma does not 'fail' if $r = 0$. The second result is new and shows that Shephard's lemma necessarily fails as long as $r > 0$ and $T > 1$, but *not* if $T = 1$ (i.e., if production takes exactly one period). Indeed, whenever $T = 1$, we have that K^d in (C.3) equals

$$K^d = \frac{w\ell_0\left[(1+r)^1 - 1\right] + w\ell_1\left[(1+r)^0 - 1\right]}{r} = w\ell_0,$$

while $\hat{K}^d$ in (C.4) equals

$$\hat{K}^d = \frac{w\ell_0\,(1+r)}{1+r} + (1-1)\,\frac{w\ell_1\,(1+r)^{1-1}}{1+r} = w\ell_0,$$

and thus the derivative of the cost function indeed delivers capital demand.

Intuition and Taking Stock What is the intuition for these results? Remember that the failure of Shephard's lemma is associated with the heterogeneous nature of physical capital goods in the model. These goods have different prices which are shaped differently by changes in the interest rates, depending on their level of completion. As pointed out in the main text, when $r \to 0$, capital remuneration vanishes, and all capital goods have the same price. And when time is discrete and production takes exactly one period in all sectors, then there is again a single type of homogeneous physical capital goods (goods one period from completion), and thus the heterogeneous nature of capital goods again vanishes. The latter is an interesting result that is obscured by our continuous time formulation, but all in all, we prefer our baseline formulation since it delivers simpler formulae for some of the key equilibrium objects of our model.

C.2 Constant Labor Use

In this Appendix, we consider an extension of the Wicksell point-input, output model in Section 3, in which labor is employed at a uniform rate along the whole production process (instead of only at its beginning). We show that the equilibrium of this model, when assuming a constant-elasticity productivity function $Z_v(T_v)$, is essentially isomorphic to the baseline Wicksell-Swan model with labor only used at the beginning of production.

Suppose that, in sector v, an amount of labor $\bar{\ell}_v$ is used at all instants from 0 to T_v. The problem faced by producers in sector v is to choose T_v and $\bar{\ell}_v$ to solve:

$$\max_{T_v}\ \pi_v = p_v Z_v(T_v)\,\bar{\ell}_v e^{-rT_v} - w\bar{\ell}_v \int_0^{T_v} e^{-rt}dt.$$

The first-order condition of this problem (after imposing zero profits) can be rewritten as

$$\frac{Z_v'(T_v)}{Z_v(T_v)} = \frac{r}{1-e^{-rT_v}}, \tag{C.5}$$

which is a slightly modified version of the 'Jevons' equation in (16). Note also that equation (C.5) is a special case of the more general optimality condition (5) because $\varepsilon_{Z_v,T_v} = Z_v'(T_v)\, T_v / Z_v(T_v)$, the function $F_v(\boldsymbol{L}_v)$ does not depend on T directly ($\varepsilon_{Y,T} = 0$), and because (imposing zero profits) $\alpha_{T_v} = e^{-rT_v} / \int_0^{T_v} e^{-rt} dt$.

In the 'Swan' version of the model with a constant-elasticity function $Z_v(T_v) = (T_v)^{\zeta_v}$, we obtain

$$T_v^* = \frac{\tilde{\zeta}_v}{r},$$

where $\tilde{\zeta}_v$ is a monotonically increasing transformation of ζ_v. More specifically, $\tilde{\zeta}_v$ is implicitly defined by

$$\zeta_v = \frac{\tilde{\zeta}_v}{1 - e^{-\tilde{\zeta}_v}},$$

where the right-hand-side is increasing in $\tilde{\zeta}_v$, implying that $\tilde{\zeta}_v$ is monotonically increasing in ζ_v. For technical reasons, we now need to assume $\zeta_v > 1$. Note that rT_v^* is independent of the interest rate, just as in the Wicksell-Swan lumber model, which again will dramatically simplify the general equilibrium of the model.

With a uniform use of labor, equation (8) in the main text simplifies to

$$\Lambda_v(r) = \beta_v \frac{rT_v}{e^{rT_v} - 1} = \beta_v \frac{\tilde{\zeta}_v}{e^{\tilde{\zeta}_v} - 1},$$

and thus the allocation labor across sectors is given by

$$L_v = \frac{\beta_v \tilde{\zeta}_v / \left(e^{\tilde{\zeta}_v} - 1\right)}{\int_0^1 \beta_v \tilde{\zeta}_v / \left(e^{\tilde{\zeta}_v} - 1\right) dv} L, \tag{C.6}$$

and is thus again independent of the interest rate, while capital intensity is given by

$$\frac{K_v^d}{L_v} = \frac{w}{r} \frac{e^{\tilde{\zeta}_v} - 1 - \tilde{\zeta}_v}{\tilde{\zeta}_v}. \tag{C.7}$$

As in our baseline specification, capital intensity is decreasing in the interest rate and monotonically decreasing in v. Imposing capital-market clearing, it is again easy to solve for the equilibrium interest rate r, as in the derivation of equation (19) in the main text.

The $\mathcal{APP}$ in sector v is given by

$$\mathcal{APP}_v(\boldsymbol{L}) = w\ell_v(t) \int_0^{T_v} (T_v - t) \frac{e^{r(T_v - t)}.}{w\bar{\ell}_v \int_0^{T_v} e^{-rt} dt} dt = \frac{e^{-rT_v} - 1 + rT_v}{r\left(1 - e^{-rT_v}\right)},$$

which plugging in $T_v^* = \tilde{\zeta}_v / r$, delivers

$$\mathcal{APP}_v(\boldsymbol{L}) = \frac{e^{-\tilde{\zeta}_v} - 1 + \tilde{\zeta}_v}{r\left(1 - e^{-\tilde{\zeta}_v}\right)},$$

which is increasing in $\tilde{\zeta}_v$ (and ζ_v) and decreasing in the interest rate, and which can actually be reduced to

$$\mathcal{APP}_v(\boldsymbol{L}) = \frac{\zeta_v - 1}{r}.$$

Next, the zero profit condition implies

$$p_v = \frac{\int_0^{T_v} e^{-rt} dt}{Z_v(T_v)\, e^{-rT_v}} w = \frac{\tilde{\zeta}_v - 1}{\left(\tilde{\zeta}_v\right)^{\zeta_v}} (r)^{\zeta_v - 1} w,$$

and thus

$$\frac{\partial \ln p_v}{\partial r} = \frac{\zeta_v - 1}{r} = \mathcal{APP}_v,$$

as predicted by Proposition 1. This indicates that comparative advantage continues to be shaped by the interaction of $\mathcal{APP}$ differences across countries and interest-rate differences across countries, and thus Proposition 3 naturally continues to hold with a uniform use of labor.

The corresponding timeless representation is

$$y_v = \left(\tilde{K}_v\right)^{1 - \frac{\tilde{\zeta}_v}{e^{\tilde{\zeta}_v} - 1}} (L_v)^{\frac{\tilde{\zeta}_v}{e^{\tilde{\zeta}_v} - 1}},$$

where labor L_v commands a wage w in all sectors, and where $\tilde{K}_v$ is a sectoral stock of physical capital. This stock can be computed by deflating the stock of financial capital K_v in sector v by a sector-specific price of a capital p_v^k which can be backed out from the following equation:

$$p_v = \frac{w}{\left(\frac{\tilde{\zeta}_v}{r}\right)^{\zeta_v}} \frac{e^{\tilde{\zeta}_v} - 1}{r} = \left(\frac{r p_v^k}{1 - \frac{\tilde{\zeta}_v}{e^{\tilde{\zeta}_v} - 1}}\right)^{1 - \frac{\tilde{\zeta}_v}{e^{\tilde{\zeta}_v} - 1}} \left(\frac{w}{\frac{\tilde{\zeta}_v}{e^{\tilde{\zeta}_v} - 1}}\right)^{\frac{\tilde{\zeta}_v}{e^{\tilde{\zeta}_v} - 1}} = C_v\left(r p_v^k, w\right).$$

C.3 Fixed Capital Supply

In this Appendix, we develop a variant of our model with a fixed supply of capital. We then revisit our main general-equilibrium results when interest rates are set under this fixed capital supply.

A Microfoundation Time is discrete and runs indefinitely: $t = 0, 1, 2, \ldots$ At any point, the economy is populated by a measure of finitely-lived individuals, who live for exactly two periods:

young and old age. When young, agents work, consume an amount c_{young} of the numéraire good, and save the rest of their income. When they are old, they just run down their assets by consuming an amount c_{old}. In any period, there is a unit mass of young individuals and a unit mass of old individuals. Preferences are given by

$$U = c_{young} + \frac{1}{\rho} \log \left(c_{old} \right),$$

where ρ governs their impatience, as in our baseline model. Agents maximize U subject to two constraints:

$$\begin{aligned} c_{young} &\leqslant wL - s \\ c_2 &\leqslant (1+r)\,\theta s. \end{aligned}$$

In these constraints, w is the wage rate, L is the labor supply of each of the young individuals, s are individual savings, r is the interest rate, and $\theta < 1$ is a measure of the quality of the financial system, measuring a sort of 'iceberg' costs associated with channeling funds from savers to firms. The intertemporal budget constraint of agents is thus

$$c_{young} + \frac{c_{old}}{(1+r)\,\theta} \leqslant wL.$$

The optimal choices of consumption when young and old are given by

$$\begin{aligned} c_{young} &= wL - \frac{1}{\rho} \\ c_{old} &= \frac{(1+r)\,\theta}{\rho}, \end{aligned}$$

which implies that aggregate savings by the young are given by

$$s = wL - c_{young} = \frac{1}{\rho},$$

and the supply of capital is given by

$$K^s = \frac{\theta}{\rho},$$

and independent of the wage rate and the interest rate. Although our microfoundation has invoked an OLG framework in discrete time, we now revert to our continuous-time baseline model, and simply assume that capital supply is given by $K^s = \theta/\rho$.

Closed Economy Equilibrium The equilibrium of a closed economy is analogous to that in the baseline model, except that the interest rate is now pinned down by equations (11) and (14), or

$$\frac{K^d}{L}=\int_0^1\frac{L_v}{L}\frac{K_v^d}{L_v}dv=\int_0^1\int_0^{T_v}\frac{w\ell_v(t)}{L}\frac{e^{r(T_v-t)}-1}{r}dtdv=\frac{\theta}{\rho L}=\frac{K^s}{L}.$$

In this equation, the wage rate w can still be expressed as a function of the interest rate r (see equation (9), and thus the above equation pins down the interest rate r. Whenever capital intensity is declining in the interest rate, and the allocation of labor is not too responsive to the interest rate, this will lead to a unique equilibrium interest rate r, and this interest rate will be declining in the aggregate capital-labor ratio K^s/L. For example, in Wicksell's timber model, we have

$$\frac{r}{w}=\frac{L}{K^s}\int_0^1\beta_v\left(1-e^{-\zeta_v}\right)dv.$$

If two countries, Home and Foreign, differ only in their capital labor ratio, it is then clear that $K^H/L^H>K^F/L^F$ implies that Home will feature a lower interest rate and a higher wage rate than Foreign under autarky.

Free Trade Equilibrium The free trade equilibrium is analogous to the one in our baseline model, except for equations (28) and (29), which are now replaced by

$$\int_0^{v*}\frac{L_v^H}{L^H}\times\frac{K_v^H}{L_v^H}dv=\frac{K^H}{L^H}=\frac{\theta^H}{\rho^H L^H}\tag{C.8}$$

and

$$\int_{v*}^1\frac{L_v^F}{L^F}\times\frac{K_v^F}{L_v^F}dv=\frac{K^F}{L^F}=\frac{\theta^F}{\rho^F L^F}.\tag{C.9}$$

It is then straightforward to show that Proposition 4, applying to *regular economies*, continues to apply in this case, precisely because it is stated in terms of equilibrium aggregate capital-labor ratios.

Factor Price Equalization Equilibria Consider then an integrated equilibrium in which capital and labor could freely move across countries. The total endowments of capital and labor are given by $\bar{L}=L^H+L^F$, and $\bar{K}=K^H+K^F$, where $\bar{w}$ and $\bar{r}$ are the equilibrium factor prices in the integrated equilibrium. These can easily be computed from the closed-economy of our model from equation (9), i.e.,

$$\ln\bar{w}=\int_0^1\beta_v\left(\zeta_v\ln\left(\zeta_v/\bar{r}\right)-\zeta_v-\ln\beta_v\right)dv,$$

and from equilibrium in the world capital market, or

$$\frac{\int_0^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^1 \beta_v e^{-\zeta_v} dv} \frac{\bar{w}}{\bar{r}} = \frac{\bar{K}}{\bar{L}}. \tag{C.10}$$

For this to be an equilibrium, the Home and Foreign countries need to allocate capital and labor across sectors in a way that clears those local markets at the integrated factor prices $\bar{w}$ and $\bar{r}$. Define the allocation of capital of Home and Foreign in sector v by K_v^H and K_v^F and the analogous allocations for labor as L_v^H and L_v^F.

For the Home capital market to clear, we need

$$\frac{\int_0^1 K_v^j dv}{L^j} = \int_0^1 \frac{L_v^j}{L^j} \frac{1 - e^{-\zeta_v}}{e^{-\zeta_v}} \frac{\bar{w}}{\bar{r}} dv = \frac{K^j}{L^j}, \tag{C.11}$$

for $j = H, F$. An equilibrium with FPE needs to satisfy the following additional constraints:

$$\begin{aligned} \int_0^1 \frac{L_v^j}{L^j} dv &= 1 \\ \int_0^1 \frac{L_v^j}{L^j} dv &= 1 \\ \frac{L_v^H + L_v^F}{L^H + L^F} &= \frac{\beta_v e^{-\zeta_v}}{\int_0^1 \beta_v e^{-\zeta_v} dv}. \end{aligned}$$

It is well-known that there are a continuum of possible allocations L_v^H and L_v^F satisfying these conditions. It is also well-known, and easily verified, that whenever $K^H/L^H, = K^F/L^F$, an equilibrium with FPE will necessarily exist. Just set

$$L_v^j = \frac{\beta_v e^{-\zeta_v}}{\int_0^1 \beta_v e^{-\zeta_v} dv} L^j \text{ for } j = H, F.$$

and verify that all the conditions for an FPE are satisfied.

Equilibria with FPE exist for asymmetric capital-labor ratios, and the maximal asymmetry consistent with FPE can be computed by studying the case in which Home specializes in the most capital intensive sectors ($v < v^*$) while Foreign specializes in the least capital intensive goods, and still factor markets clear at $\bar{w}$ and $\bar{r}$. This corresponds to the so-called 'lens condition' in Deardorff (1994), and delivers

$$\begin{aligned} L_v^H &= \frac{\beta_v e^{-\zeta_v}}{\int_0^{v*} \beta_v e^{-\zeta_v} dv} L^H \text{ for } v \leqslant v^*;\ L_v^H = 0 \text{ otherwise;} \\ L_v^F &= \frac{\beta_v e^{-\zeta_v}}{\int_{v*}^1 \beta_v e^{-\zeta_v} dv} L^F \text{ for } v > v^*;\ L_v^F = 0 \text{ otherwise,} \end{aligned}$$

with the threshold v^* given by

$$\frac{L^H}{L^H + L^F} = \frac{\int_0^{v^*} \beta_v e^{-\zeta_v} dv}{\int_0^1 \beta_v e^{-\zeta_v} dv}. \tag{C.12}$$

Whenever $K^H/L^H > K^F/L^F$, FPE is attained whenever the Home and Foreign capital-labor ratio fall within this FPE lens, or

$$\frac{\int_0^{v^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{v^*} \beta_v e^{-\zeta_v} dv} \frac{\bar{w}}{\bar{r}} \geqslant \frac{K^H}{L^H}$$

and

$$\frac{\int_{v^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_{v^*}^1 \beta_v e^{-\zeta_v} dv} \frac{\bar{w}}{\bar{r}} \leqslant \frac{K^F}{L^F},$$

where v^* is given by (C.12) and $\bar{w}/\bar{r}$ in (C.10).

Conversely, whenever $K^H/L^H < K^F/L^F$, we can define an alternative threshold $v^{*\prime}$ such that

$$\frac{L^F}{L^H + L^F} = \frac{\int_0^{v^{*\prime}} \beta_v e^{-\zeta_v} dv}{\int_0^1 \beta_v e^{-\zeta_v} dv}. \tag{C.13}$$

On Uniqueness With fixed capital supplies, and as shown above, a unique *allocation* need not obtain inside the FPE set because a continuum of specialization patterns can be consistent with market clearing. Nevertheless, with fixed capital supplies we can show uniqueness of the *complete-specialization equilibrium* outside the FPE region, at least under some specific specifications of the time-phased nature of production.

To illustrate this, consider the Wicksell–Swan. In any complete-specialization equilibrium with $r_H < r_F$ and cutoff $v^* \in (0,1)$, the labor-market and zero-profit conditions imply a strictly decreasing schedule $g(v^*)$ mapping r_H/r_F into v^*. At the same time, the two capital-market clearing conditions with exogenous (K^H, L^H) and (K^F, L^F) imply a strictly increasing schedule $f(v^*)$ mapping r^H/r^F into v^*, or

$$\frac{r^H}{r^F} = \frac{\int_0^{v^*} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_{v^*}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv} \frac{K^F}{K^H} = f\left(v^*\right). \tag{C.14}$$

This expression can be easily derived from equations (30), (C.8) and (C.9). It is trivial to see that the right-hand-side is increasing in v^*, as stated above. In sum, when the curves $g(v^*)$ and $f(v^*)$ intersect at $r_H < r_F$, they can only do so once.

When $r_H > r_F$, Foreign specializes in the low-v indexed goods, and we can follow the exact same steps and define a threshold $v^{*\prime}$ such that equilibrium at Home requires

$$\frac{r^H}{r^F} = \frac{\int_{v^{*\prime}}^{1} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{v^{*\prime}} \beta_v \left(1 - e^{-\zeta_v}\right) dv} \frac{K^F}{K^H} = \tilde{f}\left(v^*\right).$$

The schedule $\tilde{f}(v^{*\prime})$ is now *decreasing* in $v^{*\prime}$, but the labor-market and zero-profit conditions now imply a strictly *increasing* schedule $\tilde{g}(v^*)$ mapping r^H/r^F into $v^{*\prime}$, so again a unique intersection is ensured whenever $r_H > r_F$.

Finally, as in the proof of Proposition 5 in Appendix A.5, it is straightforward to verify that the conditions ensuring an equilibrium with FPE and ensuring an equilibrium without FPE are identical but with a reverse sign, and thus jointly cover the entire parameter space, thus ensuring that an equilibrium necessarily exists, and it is unique outside the FPE set.

Rather than repeating the derivations in Appendix A.5, showing the complementarity of those conditions, we next develop an alternative way to characterize the equilibrium which was suggested by an anonymous referee. Define a function $\kappa\left(r^H, r^F\right)$ such that the capital market at Home is in equilibrium (after imposing equilibrium in the other markets). One can verify that one can always implicitly define this function, but in the Wicksell–Swan model, we can actually express it as a function of the ratio of interest rates $\Delta = r^H/r^F$. More specifically, using the derivations above, we can express $\kappa\left(\Delta\right)$ as the following continuous function:

$$\kappa\left(\Delta\right) = \begin{cases} \dfrac{\int_0^{v^*(\Delta)} \beta_v\left(1-e^{-\zeta_v}\right)dv}{\int_{v^*(\Delta)}^{1} \beta_v\left(1-e^{-\zeta_v}\right)dv}\dfrac{1}{\Delta} & \text{if } \Delta < 1 \\ \left[\underline{\kappa}, \overline{\kappa}\right] = \left[\dfrac{\int_{\tilde{v}^*(1)}^{1} \beta_v\left(1-e^{-\zeta_v}\right)dv}{\int_0^{\tilde{v}^*(1)} \beta_v\left(1-e^{-\zeta_v}\right)dv}, \dfrac{\int_0^{v^*(1)} \beta_v\left(1-e^{-\zeta_v}\right)dv}{\int_{v^*(1)}^{1} \beta_v\left(1-e^{-\zeta_v}\right)dv}\right] & \text{if } \Delta = 1, \\ \dfrac{\int_{\tilde{v}^*(\Delta)}^{1} \beta_v\left(1-e^{-\zeta_v}\right)dv}{\int_0^{\tilde{v}^*(\Delta)} \beta_v\left(1-e^{-\zeta_v}\right)dv}\dfrac{1}{\Delta} & \text{if } \Delta > 1 \end{cases} \tag{C.15}$$

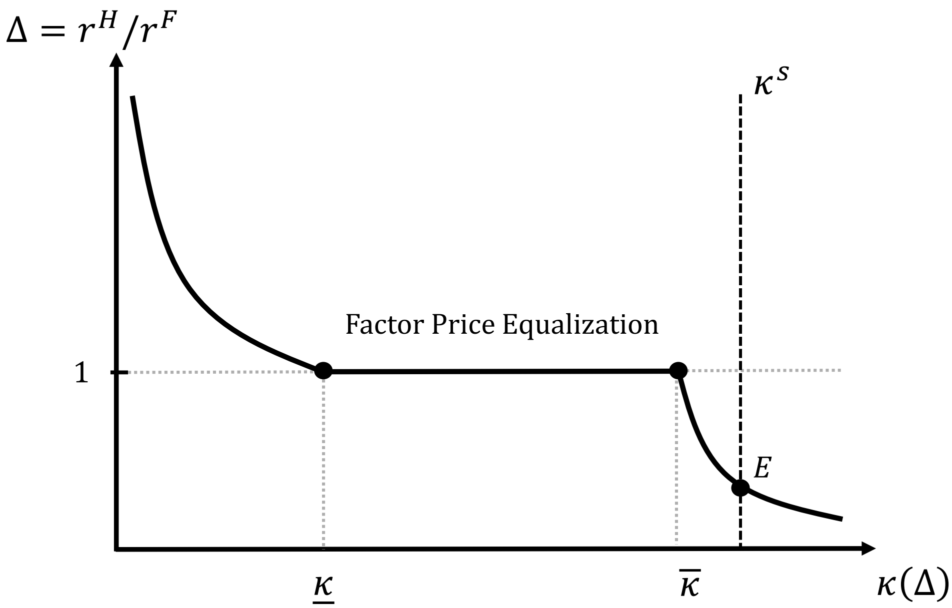
where $v^*\left(\Delta\right)$ is implicitly defined by

$$\Delta = \left(\frac{\int_0^{v^*(\Delta)} \beta_v e^{-\zeta_v} dv}{\int_{v^*(\Delta)}^{1} \beta_v e^{-\zeta_v} dv} \frac{L^F}{L^H}\right)^{-1/\zeta_{v^*(\Delta)}},$$

and $\tilde{v}^*\left(\Delta\right)$ is implicitly defined by

$$\Delta = \left(\frac{\int_{\tilde{v}^*(\Delta)}^{1} \beta_v e^{-\zeta_v} dv}{\int_0^{\tilde{v}^*(\Delta)} \beta_v e^{-\zeta_v} dv} \frac{L^F}{L^H}\right)^{-1/\zeta_{\tilde{v}^*(\Delta)}}.$$

Figure C.1: Equilibria in the Wicksell-Swan Model with Fixed Capital Supply



Notes: The figures plot the equilibrium in the Wicksell-Swan economy with the use of the function $\kappa(\Delta)$ defined in the text.

In an equilibrium with FPE, $w^H/w^F = 1$ and equation (30) implies

$$\frac{\int_0^{v^*(\Delta)} \beta_v e^{-\zeta_v} dv}{\int_{v^*(\Delta)}^1 \beta_v e^{-\zeta_v} dv} = \frac{\int_{\tilde{v}^*(\Delta)}^1 \beta_v e^{-\zeta_v} dv}{\int_0^{\tilde{v}^*(\Delta)} \beta_v e^{-\zeta_v} dv}.$$

This in turn implies that

$$\underline{\kappa} = \frac{\int_{\tilde{v}^*(1)}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_0^{\tilde{v}^*(1)} \beta_v \left(1 - e^{-\zeta_v}\right) dv} < \frac{\int_0^{v^*(1)} \beta_v \left(1 - e^{-\zeta_v}\right) dv}{\int_{v^*(1)}^1 \beta_v \left(1 - e^{-\zeta_v}\right) dv} = \overline{\kappa},$$

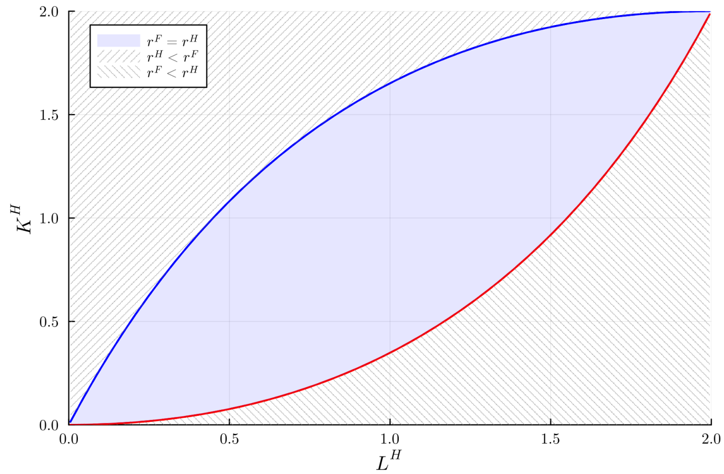
because we can alternatively write this inequality as

$$\frac{\int_{\tilde{v}^*(1)}^1 \frac{L_v^H}{L^H} \frac{1-e^{-\zeta_v}}{e^{-\zeta_v}} dv}{\int_0^{\tilde{v}^*(1)} \frac{L_v^H}{L^H} \frac{1-e^{-\zeta_v}}{e^{-\zeta_v}} dv} < 1 < \frac{\int_0^{v^*(1)} \frac{L_v^H}{L^H} \frac{1-e^{-\zeta_v}}{e^{-\zeta_v}} dv}{\int_{\tilde{v}^*(1)}^1 \frac{L_v^F}{L^F} \frac{1-e^{-\zeta_v}}{e^{-\zeta_v}} dv},$$

where the inequalities are ensured by the fact that capital intensity is monotonically declining in v.

Figure C.1 depicts this schedule $\kappa(\Delta)$. Together with a vertical capital supply schedule, the figure illustrates the existence of a unique equilibrium. In the figure, the equilibrium results in a lower interest rate at Home than in Foreign.

Figure C.2: Equilibria in the Wicksell-Swan Model with Fixed Capital Supply



Notes: The figures plot the nature of equilibria as a function of the allocation of world capital and labor to Home and Foreign, for the special case of Wicksell-Swan economy with $\beta_v = 1$ for all v, $\zeta_v = 1 - v$, and $\bar{L} = \bar{K} = 2$.

With fixed capital supply, the nature of equilibria in regular economies can also be illustrated via a production Edgeworth box representing different allocations of worldwide capital and labor across countries. We do so in Figure C.2, which adopts the same parameterization of the Wicksell-Swan model as in Figure 3, with $\beta_v = 1$ for all v and $\zeta_v = 1 - v$.

C.4 Equilibria with Time-Versus-Capital Intensity Ranking Reversals

This Appendix provides a formal treatment of the claims in Section 4.4 of the main text. Throughout, we maintain (i) no $\mathcal{APP}$ reversals, (ii) no capital intensity reversals (in the usual sense of factor-intensity rankings changing with factor prices), and (iii) sectoral capital intensity declining in the interest rate. We first focus on the tractable Wicksell–Swan "chopping off trees" extension introduced in equation (34) in the main text, and we highlight the precise mechanism through which time-versus-capital intensity ranking reversals can generate multiple equilibria and the unconventional comparative-statics discussed in Section 4.4. We then discuss how the results generalize beyond that specific 'chopping off trees' example.

The Chopping Off Trees Economy

Technology In each sector $v \in [0, 1]$, output is produced with labor used at the beginning and at the end of the production process:

$$y_v = (T_v)^{\zeta_v} \left(\ell_v^0\right)^{\alpha_v} \left(\ell_v^T\right)^{1-\alpha_v},$$

with $\zeta_v > 0$ and $\alpha_v \in (0, 1)$. Country $j \in \{H, F\}$ faces wage w^j and interest rate r^j.

Firms choose $(T_v, \ell_v^0, \ell_v^T)$ to minimize unit cost, taking into account that (i) the revenue from a unit of output is realized at completion T_v, (ii) ℓ_v^0 is paid at the beginning of production, and (iii) ℓ_v^T is paid at completion. As shown in the main text, the optimal production length is

$$T_v^j = \frac{\zeta_v}{\alpha_v r^j}, \tag{C.16}$$

which implies an average period of production (APP) equal to

$$\mathcal{APP}_v^j = \frac{\zeta_v}{r^j}. \tag{C.17}$$

Hence, if ζ_v is decreasing in v (our maintained indexing convention), there are no $\mathcal{APP}$ reversals.

As stated in the main text, the zero-profit condition implies that the unit price at which country j can supply good v takes the form

$$p_v^j = \kappa_v w^j \left(r^j\right)^{\zeta_v}, \tag{C.18}$$

for a sector-specific constant κ_v that depends on (ζ_v, α_v) but is common across countries. Equation (C.18) makes clear that relative production costs are shaped by the interaction of interest-rate differences and differences in ζ_v, exactly as in the Wicksell–Swan case studied in Section 4.3.

Sectoral Labor and Capital Demands Let σ_v denote the combination of parameters governing labor demand in this environment:

$$\sigma_v \equiv (1 - \alpha_v) + \alpha_v e^{-\zeta_v/\alpha_v}. \tag{C.19}$$

Under Cobb–Douglas preferences across goods, goods-market clearing and (C.18) imply that labor allocated to sector v in a country producing that good is proportional to $\beta_v \sigma_v$ and is independent of the interest rate. When $\alpha_v = 1$ for all v, we obtain $\sigma_v = e^{-\zeta_v}$, as in the Wicksell-Swan economy (see equation (17)).

Working-capital demand in this variant of the model is driven by the need to finance the beginning-of-process labor input ℓ_{0v} over the production interval. The associated capital demand per unit of labor (conditional on producing good v in country j) can be written (see equation (35)) as

$$\frac{K_v^j}{L_v^j} = \frac{w^j}{r^j}\mu_v, \qquad \text{where} \qquad \mu_v \equiv \frac{\alpha_v \left(1 - e^{-\zeta_v/\alpha_v}\right)}{\sigma_v}. \tag{C.20}$$

Importantly, μ_v depends on (ζ_v, α_v) but not on (w^j, r^j). As a result, there are no *capital intensity* *reversals* in the standard sense (rankings do not change with factor prices). However, μ_v need not be monotone in v even if ζ_v is monotone: this is precisely the possibility of time-versus-capital intensity *ranking* reversals emphasized in Definition 2 in the main text.

Complete-Specialization Equilibrium Conditions Consider a candidate equilibrium with factor-price differences and complete specialization, in which Home produces goods $v \in [0, v^*]$ and Foreign produces goods $v \in (v^*, 1]$, with $r_H < r_F$. Cost equalization for the marginal good v^* and (C.18) imply

$$\frac{w^H}{w^F} = \left(\frac{r^F}{r^H}\right)^{\zeta_{v^*}}. \tag{C.21}$$

Labor-market clearing implies

$$\frac{w^H}{w^F} = \frac{\int_0^{v^*} \beta_v \sigma_v dv}{\int_{v^*}^1 \beta_v \sigma_v dv} \frac{L^F}{L^H} \equiv B(v^*). \tag{C.22}$$

Combining (C.21)–(C.22) yields a schedule mapping v^* into the interest-rate ratio:

$$\frac{r^H}{r^F} = (B(v^*))^{-1/\zeta_{v^*}} \equiv g(v^*). \tag{C.23}$$

Under our maintained assumption that ζ_v is decreasing in v, $B(v^*)$ is increasing in v^*, and thus $g(v^*)$ is continuous and strictly decreasing on $(0, 1)$ (the same logic as in Appendix A.5).

Aggregating (C.20) over the set of goods produced by j and using the fact that sectoral labor shares are proportional to $\beta_v \sigma_v$, total capital demand satisfies

$$K^{jd} = \frac{w^j L^j}{r^j} \bar{\mu}^j (v^*), \qquad \bar{\mu}^H (v^*) \equiv \frac{\int_0^{v^*} \beta_v \sigma_v \mu_v dv}{\int_0^{v^*} \beta_v \sigma_v dv}, \qquad \bar{\mu}^F (v^*) \equiv \frac{\int_{v^*}^1 \beta_v \sigma_v \mu_v dv}{\int_{v^*}^1 \beta_v \sigma_v dv}. \tag{C.24}$$

Plugging in (C.22), we can write the capital-market clearing condition as follows:

$$\kappa (\Delta) = \frac{\int_0^{v^*(\Delta)} \beta_v \sigma_v \mu_v dv}{\int_{v^*(\Delta)}^1 \beta_v \sigma_v \mu_v dv} \frac{1}{\Delta} = \frac{K^H}{K^F}, \tag{C.25}$$

where $\Delta = r^H/r^F$. Because $v^* (\Delta)$ is decreasing in Δ, the schedule $\kappa (\Delta)$ is also decreasing in Δ, and is completely analogous to the part of the schedule (C.15) applying when $\Delta < 1$ developed in Appendix C.3 for the Wicksell-Swan model. In fact, for $\alpha_v = 1$ for all v, we obtain the exact same schedule.

Similarly, when $\Delta > 1$, we can easily derive that

$$\kappa(\Delta) = \frac{\int_{\tilde{v}^*(\Delta)}^{1} \beta_v \sigma_v \mu_v dv}{\int_0^{\tilde{v}^*(\Delta)} \beta_v \sigma_v \mu_v dv} \frac{1}{\Delta} = \frac{K^H}{K^F},$$

where again $\Delta < 1$.

Factor-Price Equalization Equilibria Let $\Delta = 1$ denote factor-price equalization. In that case, $r_H = r_F = r^*$, with r^* pinned down by world capital-market clearing. At $\Delta = 1$, production patterns are indeterminate: many allocations of sectoral production across countries are consistent with common factor prices. As a result, the set of relative capital demand ratios that can be supported under FPE is an interval

$$\kappa(1) = [\underline{\kappa}, \overline{\kappa}],$$

where $(\underline{\kappa}, \overline{\kappa})$ solve a simple linear program (min/max of Home capital intensity over feasible allocations), and where the bounds $\underline{\kappa}$ and $\overline{\kappa}$ are given by:

$$\underline{\kappa} = \frac{\int_{\tilde{\omega}^*}^{1} \beta_\omega \sigma_\omega \mu_\omega d\omega}{\int_0^{\tilde{\omega}^*} \beta_\omega \sigma_\omega \mu_\omega d\omega}, \tag{C.26}$$

and

$$\bar{\kappa} = \frac{\int_0^{\omega^*} \beta_\omega \sigma_\omega \mu_\omega d\omega}{\int_{\omega^*}^{1} \beta_\omega \sigma_\omega \mu_\omega d\omega}, \tag{C.27}$$

respectively. Crucially, in these expressions, sectors are indexed in decreasing order of capital intensity, or such that μ_ω in equation (C.24) is decreasing in ω. Furthermore, the limit thresholds ω^* and $\tilde{\omega}^*$ are defined by

$$1 = \left(\frac{\int_0^{\omega^*} \beta_\omega \sigma_\omega d\omega}{\int_{\omega^*}^{1} \beta_\omega \sigma_\omega d\omega} \frac{L^F}{L^H} \right)^{-1/\zeta_{\omega^*}}$$

and

$$1 = \left(\frac{\int_{\tilde{\omega}^*}^{1} \beta_\omega \sigma_\omega d\omega}{\int_0^{\tilde{\omega}^*} \beta_\omega \sigma_\omega d\omega} \frac{L^F}{L^H} \right)^{-1/\zeta_{\tilde{\omega}^*}}$$

respectively.

These two implicit conditions imply

$$\frac{\int_0^{\omega^*} \beta_\omega \sigma_\omega d\omega}{\int_{\omega^*}^{1} \beta_\omega \sigma_\omega d\omega} = \frac{\int_{\tilde{\omega}^*}^{1} \beta_\omega \sigma_\omega d\omega}{\int_0^{\tilde{\omega}^*} \beta_\omega \sigma_\omega d\omega},$$

which allows us to write the condition $\underline{\kappa} < \bar{\kappa}$ as

$$\frac{\frac{\int_{\tilde{\omega}*}^{1}\beta_\omega\sigma_\omega\mu_\omega d\omega}{\int_{\tilde{\omega}*}^{1}\beta_\omega\sigma_\omega d\omega}}{\frac{\int_0^{\tilde{\omega}*}\beta_\omega\sigma_\omega\mu_\omega d\omega}{\int_0^{\tilde{\omega}*}\beta_\omega\sigma_\omega d\omega}} < 1 < \frac{\frac{\int_0^{\omega*}\beta_\omega\sigma_\omega\mu_\omega d\omega}{\int_0^{\omega*}\beta_\omega\sigma_\omega d\omega}}{\frac{\int_{\omega*}^{1}\beta_\omega\sigma_\omega\mu_\omega d\omega}{\int_{\omega*}^{1}\beta_\omega\sigma_\omega d\omega}},$$

where the inequalities are ensured by the fact that μ_ω is monotonically decreasing in ω by construction. In sum, we necessarily have $\underline{\kappa} < \bar{\kappa}$.

Multiplicity of Equilibria Overall, we have shown above that we can reduce the equilibrium of the model to a relative interest rate $r^H/r^F = \Delta$ that equates relative capital supply K^H/K^F with the relative capital demand schedule

$$\kappa(\Delta) = \begin{cases} \frac{\int_0^{v*(\Delta)}\beta_v\sigma_v\mu_v dv}{\int_{v*(\Delta)}^{1}\beta_v\sigma_v\mu_v dv}\frac{1}{\Delta} & \text{if } \Delta < 1 \\ [\underline{\kappa}, \overline{\kappa}] & \text{if } \Delta = 1 \\ \frac{\int_{\tilde{v}*(\Delta)}^{1}\beta_v\sigma_v\mu_v dv}{\int_0^{\tilde{v}*(\Delta)}\beta_v\sigma_v\mu_v dv}\frac{1}{\Delta} & \text{if } \Delta > 1 \end{cases}. \tag{C.28}$$

This is analogous to equation (C.15) in the Wicksell-Swan model (see Appendix C.3), but there is an important difference. Note that as Δ converges to 1 from below we necessarily have that

$$\lim_{\Delta\to 1^-}\kappa(\Delta) = \frac{\int_0^{v*(1)}\beta_v\sigma_v\mu_v dv}{\int_{v*(1)}^{1}\beta_v\sigma_v\mu_v dv} \leqslant \bar{\kappa}, \tag{C.29}$$

with a strict inequality when the ranking of sectors according to v and ω do *not* coincide, which corresponds to the existence of time-versus-capital ranking reversals.

To prove this, note that as $w^H/w^F = 1$, we necessarily have

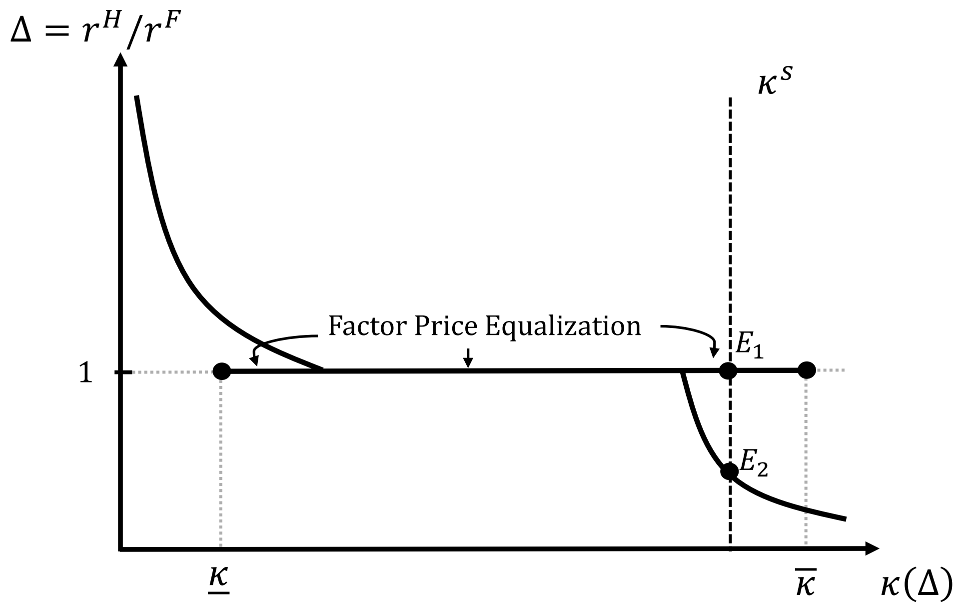
$$\frac{\int_0^{v*(1)}\beta_v\sigma_v dv}{\int_{v*(1)}^{1}\beta_v\sigma_v dv} = \frac{\int_0^{\omega*}\beta_\omega\sigma_\omega d\omega}{\int_{\omega*}^{1}\beta_\omega\sigma_\omega d\omega},$$

and thus $\lim_{\Delta\to 1^-}\kappa(\Delta) < \bar{\kappa}$ if and only if

$$\frac{\int_0^{v*(1)}\frac{L_v^H}{L^H}\mu_v dv}{\int_{v*(1)}^{1}\frac{L_v^F}{L^F}\mu_v dv} < \frac{\int_0^{\omega*}\frac{L_v^H}{L^H}\mu_v dv}{\int_{\omega*}^{1}\frac{L_v^F}{L^F}\mu_v dv}, \tag{C.30}$$

but this condition must necessarily hold because $\bar{\kappa}$ is obtained by picking precisely the feasible allocation with the highest implied aggregate capital-labor ratio demanded. In other words, the allocation implicit in the left-hand-side of (C.30) is certainly feasible in the FPE set, so if it does not correspond to the same one as in the right-hand-side of (C.30), then it must entail a

Figure C.3: Equilibria in the Wicksell-Swan Model when Chopping Off Trees with Fixed Capital Supply



Notes: The figures plot the equilibrium in the Wicksell-Swan economy with 'chopping off of trees' with the use of the function $\kappa(\Delta)$ defined in the text.

(weakly) lower implied average capital-intensity.

We can similarly show that

$$\underline{\kappa} \leqslant \lim_{\Delta \to 1^+} \kappa(\Delta) = \frac{\int_0^{v^*(1)} \beta_v \sigma_v \mu_v dv}{\int_{v^*(1)}^1 \beta_v \sigma_v \mu_v dv},$$

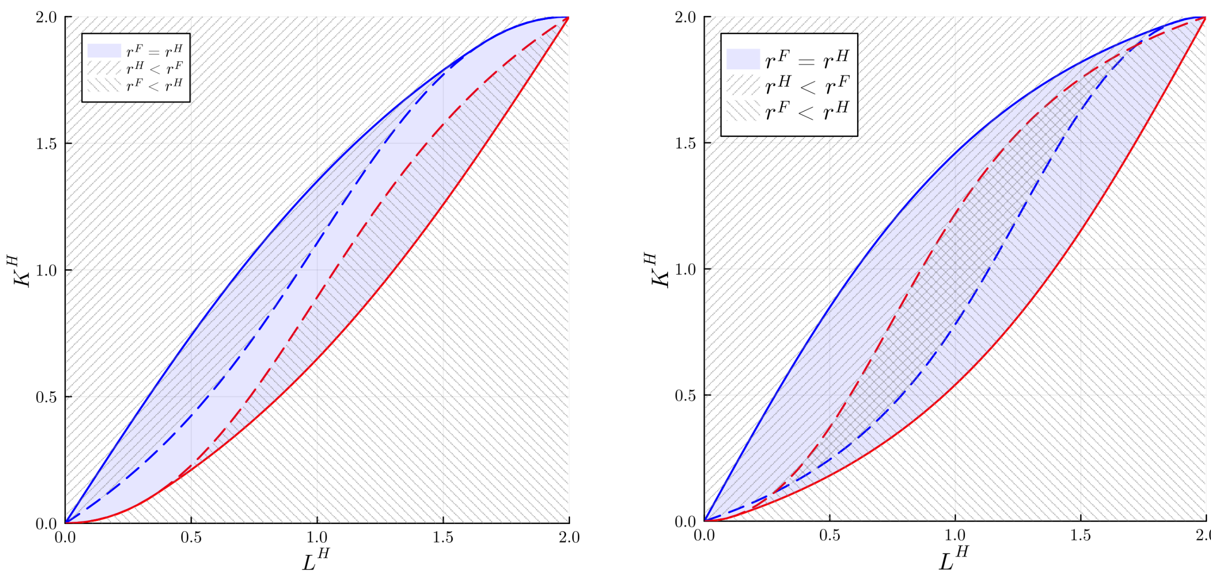
with a strict inequality when the ranking of sectors according to v and ω do *not* coincide, which again corresponds to the existence of time-versus-capital ranking reversals.

Putting all the pieces together, we have demonstrated that the schedule $\kappa(\Delta)$ in equation (C.28) is upper hemi-continuous and for a given fixed relative supply K^H/K^F of capital at Home and in Foreign, we can guarantee the existence of an equilibrium, but there certainly is the possibility of multiple equilibria. This possibility is depicted in Figure C.3 with one equilibrium featuring FPE and another one featuring a lower interest rate at Home than in Foreign.

With fixed capital supplies, the multiplicity of equilibria can also be illustrated with the more familiar production Edgeworth box representing different allocations of worldwide capital and labor across countries. Figure C.4 plots the nature of equilibria for different allocations of worldwide capital and labor across countries for two cases. The left panel is produced by assuming $\beta_v = 1$ for all, $\zeta_v = 1 - v$, and $\alpha_v = e^{-2(1-v)}$, while the right panel is obtained

Figure C.4: Equilibria in the Wicksell-Swan Model when Chopping Off Trees and Fixed Capital Supply

(a) $\alpha_v = \exp(-2 * (1 - v))$ and $\zeta_v = 1 - v$ **(b)** $\alpha_v = \exp(-3 * (1 - v))$ and $\zeta_v = 1 - v$



Notes: The figures plot the nature of equilibria as a function of the allocation of world capital and labor to Home and Foreign, for the Wicksell-Swan economy with 'chopping off of trees' where $\beta_v = 1$ for all v while ζ_v and α_v are defined in the figure headers.

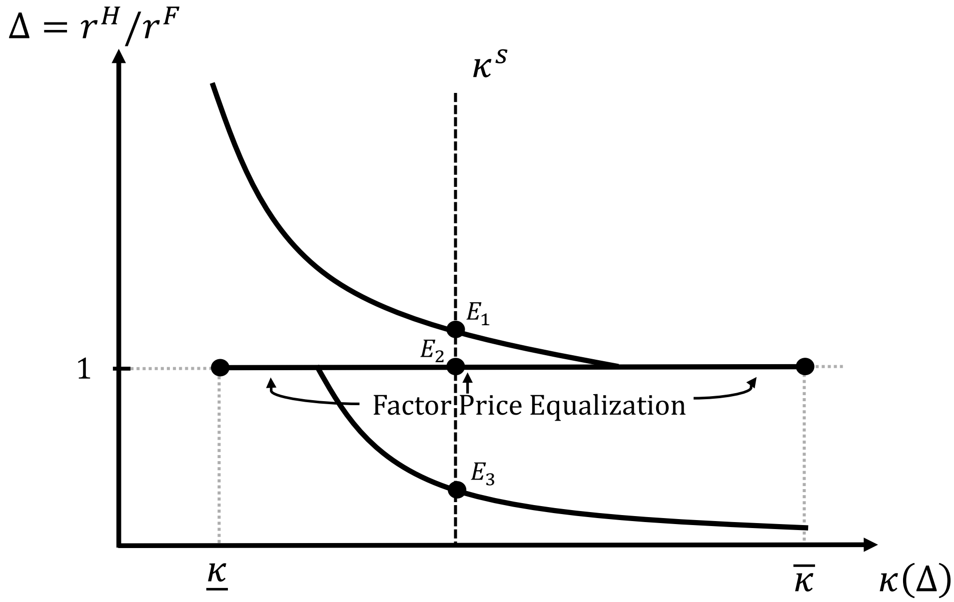
when $\beta_v = 1$ for all, $\zeta_v = 1 - v$, and $\alpha_v = e^{-3(1-v)}$. In both cases, an equilibrium with FPE always exists around the 45-degree line, and in both panels we observe the existence of multiple equilibria for some allocations within the FPE set. There are however some qualitative differences in the two graphs. In the left panel, the coexistence of FPE and non-FPE equilibria occurs when capital-labor ratios are relatively asymmetric across countries, and at most two types of equilibria exist (as in Figure C.3). In the right panel, multiplicity occurs for all endowments in the FPE set, and when countries are relatively symmetric, all three types of equilibria can exist, in line with our discussion in the main text.

The equilibrium patterns in the left panel of Figure C.4 are in line with the representation of the equilibrium via the function $\kappa(\Delta)$ in Figure C.3. The patterns in the right panel of Figure C.4 are not, however. How can one rationalize the coexistence of all three types of equilibria via the $\kappa(\Delta)$ schedule?

The key is to realize that if time-versus-capital intensity ranking reversals are sufficiently pronounced, then it is possible that

$$\lim_{\Delta \to 1^-} \kappa(\Delta) = \lim_{\Delta \to 1^-} \frac{\int_0^{v^*(\Delta)} \beta_v \sigma_v \mu_v dv}{\int_{v^*(\Delta)}^1 \beta_v \sigma_v \mu_v dv} \frac{1}{\Delta} < \lim_{\Delta \to 1^+} \frac{\int_{\tilde{v}^*(\Delta)}^1 \beta_v \sigma_v \mu_v dv}{\int_0^{\tilde{v}^*(\Delta)} \beta_v \sigma_v \mu_v dv} \frac{1}{\Delta} = \lim_{\Delta \to 1^+} \kappa(\Delta),$$

Figure C.5: Equilibria in the Wicksell-Swan Model when Chopping Off Trees with Fixed Capital Supply



Notes: The figures plot the equilibrium in the Wicksell-Swan economy with 'chopping off of trees' with the use of the function $\kappa(\Delta)$ defined in the text.

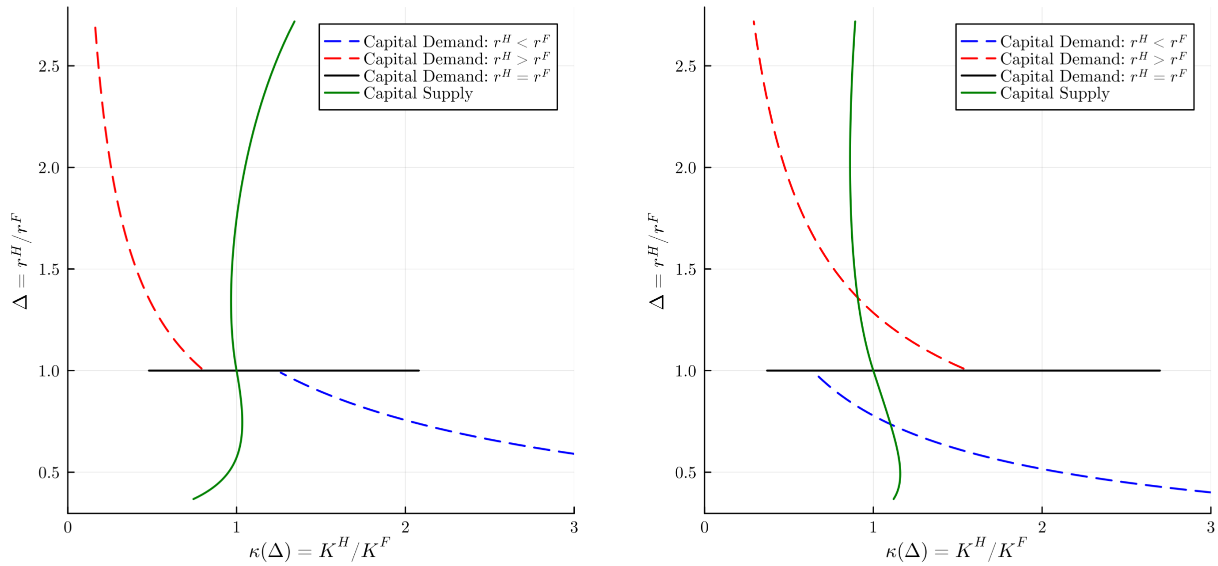
despite the fact that the term on the right of this inequality involves goods with disproportionately short $\mathcal{APP}$s. In those situations, the equilibrium equating $\kappa(\Delta)$ to relative capital supply is represented as in Figure C.5, which demonstrates the existence of the three types of equilibrium for sufficiently similar relative capital-labor ratios.

Notice that, relative to the numerical example in the left panel of Figure C.4, the numerical example in the right panel introduces an α_v function that produces a stronger positive relationship between v and α_v, and thus a higher likelihood of time-versus-capital ranking reversals. Figure C.6 provides a numerical simulation of the equilibrium in the capital market for the numerical examples in Figure C.4.

Endogenous Capital Supply In our discussion of multiplicity above, we have focused on examples with a fixed supply of capital. As mentioned in the main text, this avoids some technical difficulties associated with the fact, that when wages are endogenous, it becomes possible for the supply of capital to become backward bending for some interest rates (once the negative impact of the interest rate on wages is factored in). This implies that, in principle, one cannot rule out the existence of multiple equilibria even in regular economies without time-versus-capital ranking reversals. We have not been able to generate such multiplicity numerically in regular economies, but theoretically, the existence of multiple equilibria in that case would be captured by the pattern in Figure C.7.

Figure C.6: Equilibria in the Wicksell-Swan Model when Chopping Off Trees and Fixed Capital Supply

(a) $\alpha_v = \exp(-2*(1-v))$ and $\zeta_v = 1-v$ **(b)** $\alpha_v = \exp(-3*(1-v))$ and $\zeta_v = 1-v$



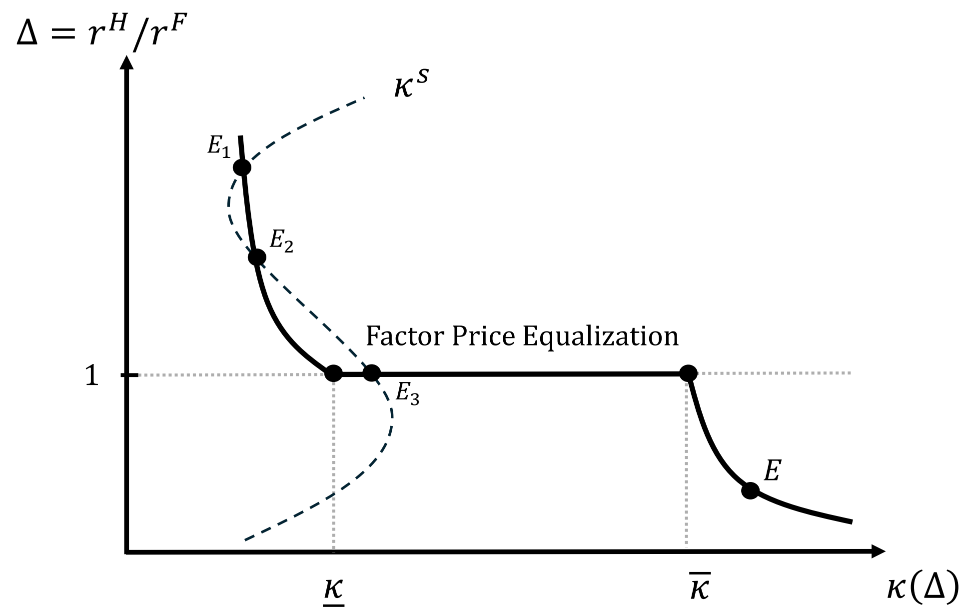
Notes: The figures plot capital demand and supply for a symmetric allocation of endowments, $L^H/L^F = \phi^F/\phi^H = 1$. The parametric model is a Wicksell-Swan economy with 'chopping off of trees' where $\beta_v = 1$ for all v while ζ_v and α_v are defined in the figure headers.

Because the source of this multiplicity is somewhat orthogonal to the novel aspects of our model, we have chosen to downplay it in the main text. But in our numerical simulations of the model, we find that the (relative) capital supply schedule is often backward bending for some range of interest rates, as seen in Figure C.6.

Generality of the Results We have demonstrated the possible existence of multiple equilibria for a version of our model with labor used at the beginning and at the end of the production process (i.e., in our 'chopping off trees' Wicksell-Swan economy). Nevertheless, as should be clear from the discussion above, the existence of time-versus-capital reversals remains an important determinant of multiplicity in more general time-phased production process. Indeed, we can always reduce the system of equations to one determining the equilibrium in the capital market (after imposing equilibrium in other markets) as the intersection of a (relative) Home capital demand schedule $\kappa(\Delta)$ and a (relative) Home capital supply schedule. In our examples above, the $\kappa(\Delta)$ schedule was well-behaved: hemi-continuous and (weakly) decreasing. For general time-phased production processes, proving such weak monotonicity is not possible. But note that the possible existence of non-monotonicities in the $\kappa(\Delta)$ schedule would only give rise to additional sources of multiplicity distinct from those emphasized in our paper.

The key novel aspect of our analysis is that, in the presence of time-versus-capital ranking

Figure C.7: Equilibria in the Wicksell-Swan Model with Endogenous Capital Supply



Notes: The figures plot the equilibrium in the Wicksell-Swan economy with endogenous capital supply with the use of the function $\kappa(\Delta)$ defined in the text.

reversals, the schedule $\kappa(\Delta)$ ceases to be continuous when hitting the FPE condition $\Delta = 1$ and becomes hemicontinuous, and this gives rise to the possibility of multiple equilibria. This is clear in our examples above, but it remains true with general time-phased production processes.

We next formally discuss some implications of the above results.

Unconventional Specialization Patterns In a complete-specialization equilibrium with $r_H < r_F$, Home produces goods $[0, v^*]$ and imports goods $(v^*, 1]$. Using (C.20), Home's (counterfactual) average physical capital intensity if it were to produce the imported goods at its own factor prices is proportional to

$$\bar{\mu}_H^{\text{imp}}(v^*) \equiv \frac{\int_{v*}^1 \beta_v \sigma_v \mu_v \, dv}{\int_{v*}^1 \beta_v \sigma_v \, dv}.$$

Home's average capital intensity in its *exports* is proportional to $\bar{\mu}_H(v^*)$. Therefore, Home exports goods that are less physically capital intensive (at Home factor prices) than the goods it imports if and only if

$$\bar{\mu}_H(v^*) < \bar{\mu}_H^{\text{imp}}(v^*). \tag{C.31}$$

Condition (C.31) cannot occur when μ_v is monotone decreasing in v, but it can occur when μ_v exhibits ranking reversals. This provides a formal statement of the second "peculiar aspect" emphasized in section 4.4 of the main text.

Unconventional Interest-Rate Ordering Suppose $\phi_H < \phi_F$ (Home is more patient / financially developed). In regular economies, the equilibrium typically features $r_H < r_F$. With ranking reversals, there may exist equilibria in which Foreign specializes in the low-v (long-$\mathcal{APP}$) goods, and thus $r_F < r_H$. Formally, such equilibria correspond to solutions of the analogue of (C.23)–(C.25) with roles reversed (Foreign producing $[0, v^*]$). When countries are symmetric (same (ϕ, L)), any asymmetric complete-specialization equilibrium has a mirror counterpart obtained by swapping country labels, so the coexistence of equilibria with $r_H < r_F$ and $r_F < r_H$ is immediate. Moreover, whenever the intersection conditions are strict, such multiple equilibria persist for small perturbations away from symmetry (by continuity), which delivers cases in which the less patient country can sustain the lower interest rate because the long-$\mathcal{APP}$ goods happen to have sufficiently low average μ_v.

Factor Content of Trade Finally, we formalize the factor-content statement in section 4.4. In any complete-specialization equilibrium, each country consumes a constant fraction of every good proportional to its income share (Cobb–Douglas demand). As a result, a constant fraction of each country's production is exported, and a constant fraction of the other country's production is imported. In particular, letting $s^H \equiv Y^H/Y^W$ denote Home's share of world income, Home's net exports of capital services (measured using the exporter input coefficients) satisfy

$$FX^{HK} = (1 - s^H)K^H - s^H K^F = K^H - s^H(K^H + K^F),$$

and similarly for labor. Hence Home is a net exporter of capital services if and only if its share of world capital exceeds its share of world income, the standard Heckscher–Ohlin–Vanek condition. This holds regardless of whether Home's export bundle is less physically capital intensive than its import bundle in the sense of equation (C.31).

C.5 Time as a Trade Cost

In this Appendix, we provide more details on the general equilibrium of our model in the presence of trade costs. Although in the main text we study separately the impact of instantaneous iceberg trade costs and of temporal trade costs, we study them jointly here.

More specifically, we assume that, relative to domestic transactions, shipping goods across borders involves an additional interval of time d between the time at which production is completed and the time at which the shipment is received and payment is made. Furthermore, we assume that shipping involves iceberg trade costs τ (shipping fees, insurance, etc.) incurred after production but before the shipment arrives at the importer's country.

Partial Equilibrium Results Our first partial-equilibrium result is that the presence of trade costs is immaterial for the endogenous determination of the average period of production or for the length of the production processes. To see this, consider the problem of producers of a given sector in country $j \in \{H, F\}$ choosing the optimal way to produce a good for a given destination market $j' \in \{H, F\}$. Ignoring sectoral time indices, the problem is

$$\begin{array}{ll} \min\limits_{\boldsymbol{L}^{jj'}, T^j} & \tau^{\mathcal{I}(j \neq j')} e^{r^j d^{\mathcal{I}(j \neq j')}} \int_0^T w^j \ell^{jj'}(t)\, e^{r^j \left(T^j - t\right)} dt \\ \text{s.t.} & y\left(\boldsymbol{L}^{jj'}\right) \geqslant 1 \end{array},$$

where $\mathcal{I}(j \neq j')$ is an indicator function taking a value of 1 if $j \neq j'$, and 0 otherwise. It is straightforward to see that the solution of this problem is independent of whether $j = j'$ (production for the domestic market) or $j \neq j'$ (production for exports).

As a result, the minimum price at which country j can produce good v and sell it in country j' is

$$p_n^{jj'} = \tau^{\mathcal{I}(j \neq j')} e^{r^j d^{\mathcal{I}(j \neq j')}} \tilde{a}_{Lv}^j \left(r^j\right) w^j = \begin{cases} \tau e^{r^j d} \tilde{a}_{Lv}^j \left(r^j\right) w^j & \text{if } j' \neq j \\ \tilde{a}_{Lv}^j \left(r^j\right) w^j & \text{if } j' = j \end{cases}.$$

Let us define $\tilde{\tau}^j \equiv \tau e^{r^j d} > 1$. Because the free trade equilibrium generically features factor price differences across countries (see below), we necessarily have that either $\tilde{\tau}^H < \tilde{\tau}^F$ or $\tilde{\tau}^H > \tilde{\tau}^F$ and thus trade costs are asymmetric and relatively lower in the country featuring a lower interest rate. Furthermore, because trade costs have a temporal dimension, reductions in interest rates worldwide tend to reduce trade costs. Both of these results were mentioned in the main text, and related to empirical patterns unveiled in the literature.

General Equilibrium Results Trade costs not only affect the location of production, but by doing so, they also affect factor markets and thus factor prices. We next outline the equations that determine the general equilibrium of the model with costly trade. As is well-known, with trade costs, factor price equalization can only occur in a set of measure zero of parameters. We can thus focus on equilibria with factor price differences.

The presence of trade costs does not invalidate Proposition 3, in the sense that the relative price of Home versus Foreign will be higher the lower is the index v (in the absence of $\mathcal{APP}$ reversals, of course), provided that $r_H < r_F$. For given interest rates , the equilibrium is in fact analogous to that in Dornbusch et al. (1977) in the presence of trade costs. More specifically, there will be two thresholds v_H^* and v_F^*, such that Home exports all goods with $v \in [0, v_H^*]$, Foreign exports all goods with $v \in [v_F^*, 1]$, and the range of goods $v \in (v_H^*, v_F^*)$ are produced in both countries and not traded. The thresholds are defined by

$$\frac{w^H}{w^F} = A\left(v_H^*\right)/\tilde{\tau}^H \tag{C.32}$$

and

$$\frac{w^H}{w^F} = A\left(v_F^*\right)\tilde{\tau}^F, \tag{C.33}$$

respectively, where remember that $A\left(v_H^*\right) = \tilde{a}_{Lv}^F\left(r^F\right)/\tilde{a}_{Lv}^H\left(r^H\right)$.

Imposing the trade-balance condition

$$\int_0^{v_H^*} \beta_v\left(w^F L^F + r^F K^F\right) = \int_{v_F^*}^1 \beta_v\left(w^H L^H + r^H K^H\right),$$

and using

$$K^j = \frac{w^j L^j}{\phi^j - r^j},$$

we obtain a condition linking the relative wage to the thresholds v_H^* and v_H^*

$$\frac{w^H}{w^F} = \frac{\int_0^{v_H^*}\beta_v}{\int_{v_F^*}^1\beta_v} \times \frac{\phi^F}{\phi^H}\frac{\phi^H - r^H}{\phi^F - r^F} \times \frac{L^F}{L^H}. \tag{C.34}$$

For given interest rates r^H and r^F, w^H/w^F is increasing in v_H^* and v_F^*. On the other hand, in equations (C.32) and (C.33), the thresholds v_H^* and v_F^* are negatively related to w^H/w^F. This implies that equation (C.34) delivers a unique equilibrium relative wage w^H/w^F for given interest rates, from which the thresholds v_H^* and v_F^* can then be solved from equations (C.32) and (C.33).

We finally turn to equilibrium in the capital market, which is now determined by

$$\int_0^{v_F^*}\frac{L_v^H}{L^H}\times\frac{K_v^H}{L_v^H}dv = \frac{w^H}{\phi^H - r^H} \tag{C.35}$$

and

$$\int_{v_H^*}^1\frac{L_v^F}{L^F}\times\frac{K_v^F}{L_v^F}dv = \frac{w^F}{\phi^F - r^F}, \tag{C.36}$$

where we need to take into account of the fact that sectors in $v \in \left(v_H^*, v_F^*\right)$ demand capital both at Home and in Foreign. Capital demand in each sector is a function of the capital demand associated with domestic and potentially foreign sales. For sectors in $v \in \left(v_H^*, v_F^*\right)$, we have

$$K_v^j = \int_0^{T_v} w^j \ell_v^j(t)\frac{e^{r^j(T_v - t)} - 1}{r^j}dt,$$

for $j = H, F$, while for $v \in (0, v_H^*)$, we have

$$K_v^j = \sum_{j' \in \{H,F\}} \int_0^{T_v} w^j \ell_v^{jj'}(t) \frac{e^{r^j\left(T_v + d^{\mathcal{I}(j \neq j')} - t\right)} - 1}{r^j} dt.$$

As in our baseline model, it is hard to obtain any general results regarding the existence and uniqueness of a general equilibrium. We note, however, that the temporal nature of trade costs, generates an additional source of capital demand, with an additional upward pressure on the interest rate for countries with lower interest rates to begin with (which other things equal, would be disproportionately more likely to export on account of their endogenously lower trade costs). Still, for large initial differences in the parameters ϕ^H and ϕ^F, it is clear that in *regular economies*, Home will feature a lower interest rate and a higher wage rate (see equations (C.35) and (C.36)). We leave a more exhaustive study of the general equilibrium of the variant of this model for future work. In the main text, we focus our discussion on the partial-equilibrium results whenever Home ends up with a lower interest rate than Foreign.

C.6 Global Value Chains

In this Appendix, we provide more details on the version of our model with global value chains (GVCs) developed in Section 6.2 of the main text. This framework is particularly closely related to the model in a previous version of this paper (Antràs, 2023a), so we refer readers to that paper for some details that will be omitted here (because they are not pertinent to the assertions in the main text of the current paper).

Environment We begin by reviewing the main assumptions of the model, which builds on the framework in Antràs and de Gortari (2020). Consumers now value a single final good, which is taken to be the numéraire, and which needs to undergo N sequential stages to be produced. At each stage $n > 1$, production combines labor with the good finished up to the previous stage $n-1$ according to

$$y_n(\boldsymbol{L}_n, \mathbf{y}_{n-1}) = (Z_n(T_n) F_n(\boldsymbol{L}_n))^{\alpha_n} (G_n(\mathbf{y}_{n-1}))^{1-\alpha_n}, \tag{C.37}$$

where y_n is output after stage n is performed. In this expression, Z_n denotes labor productivity at stage n (potentially a function of the length T_n of production), and $\alpha_n \in [0,1]$ is labor- or value-added-intensity at stage n, with $\alpha_1 = 1$. As in our baseline model, $\boldsymbol{L}_n$ is an infinite-dimensional vector of labor use along the production process, i.e., $\boldsymbol{L}_n = \{\ell_n(t)\}_{t \in [0,T_n]}$, while $\mathbf{y}_{n-1}$ is an analogous vector capturing the use of the stage $n-1$ input along the chain. Therefore, the formulation in (C.37) generalizes our assumption in the main text that the stage $n-1$

input was only purchased at the beginning of stage n production.

Production technologies in equation (C.37) are freely available to all agents in the economy, and all markets – including those for the stage-specific goods – are perfectly competitive.

Partial Equilibrium Results The total cost of production of a given stage n is given by

$$C_n\left(\boldsymbol{L}_n, \mathbf{y}_{n-1}\right) = \int_0^{T_n} \left(w\ell_n\left(t\right)e^{r(T_n-t)} + c_{n-1}y_{n-1}\left(t\right)e^{r(T_n-t)}\right)dt,$$

where c_{n-1} is the unit cost (and thus price) of stage $n-1$ output. We can then define the average period of production or $\mathcal{APP}$ in stage n as

$$\mathcal{APP}_n = \int_0^{T_n} \left(T_n - t\right) \frac{w\ell_n\left(t\right)e^{r(T_n-t)} + c_{n-1}y_{n-1}\left(t\right)e^{r(T_n-t)}}{C_n\left(\boldsymbol{L}_n, \mathbf{y}_{n-1}\right)}dt,$$

which is a weighted average temporal distance between the time $t \in [0, T_n]$ when labor *and* input expenditures are incurred and the time T_n marking the completion of stage-n production process and its sale to stage $n+1$ producers (or to consumers when $n = N$).

The path of labor expenditures $\ell_n\left(t\right)$ and input expenditures $y_{n-1}\left(t\right)$ is determined by a standard cost minimization problem

$$\begin{array}{ll} \min\limits_{\boldsymbol{L}} & \int_0^{T_n} \left(w\ell_n\left(t\right)e^{r(T_n-t)} + c_{n-1}y_{n-1}\left(t\right)e^{r(T_n-t)}\right)dt \\ \text{s.t.} & y_n\left(\boldsymbol{L}_n, \mathbf{y}_{n-1}\right) \geqslant 1 \end{array}. \tag{C.38}$$

The solution to this problem delivers a vector of unit labor requirements $\mathbf{a}_{\mathbf{L}n} = \{a_{Ln}\left(t\right)\}_{t\in[0,T_n]}$ and of unit input requirements $\mathbf{a}_{\mathbf{y}n} = \{a_{yn}\left(t\right)\}_{t\in[0,T_n]}$, with an implied unit cost function for stage n given by

$$c_n\left(w, r\right) = \int_0^{T_n} \left(wa_{Ln}\left(t\right)e^{r(T_n-t)} + c_{n-1}a_{yn}\left(t\right)e^{r(T_n-t)}\right)dt. \tag{C.39}$$

Invoking the envelope theorem, and analogously to Proposition 1 in the main text, we have

$$\frac{\partial \ln c_n\left(w, r\right)}{\partial r} = \mathcal{APP}_n. \tag{C.40}$$

Hence, the percentage response of production costs in stage n to changes in the interest rate is given precisely by the $\mathcal{APP}$ of that stage. This equation hints at the fact that, as in our baseline model, in the free trade equilibrium, comparative advantage will be shaped by the interaction of interest rate differences across countries and $\mathcal{APP}$ differences across stages. We will study this more formally below.

Closed Economy Equilibrium We now consider the general equilibrium of a closed economy with GVCs. In doing so, we revert to the assumption in the main text that stage $n-1$ input is only purchased at the beginning of stage n production. This affords great tractability because we can iterate forward the stage-specific technologies, and express aggregate final output as

$$y_N = \prod_{n=1}^{N} \left(Z_n\left(T_n\right) F_n\left(\boldsymbol{L}_n\right)\right)^{\beta_n},$$

where

$$\beta_n \equiv \alpha_n \prod_{m=n+1}^{N} \left(1-\alpha_m\right), \tag{C.41}$$

with the convention $\prod_{m=N+1}^{N}\left(1-\alpha_m\right)=1$. In words, final output is a Cobb-Douglas aggregator of the labor services of all stages of production, much like the Cobb-Douglas preferences in equation (6) of the main text. The total cost of production of this final good is in turn given by

$$c_N = \Upsilon \prod_{n=1}^{N} \left(e^{\sum\limits_{m=n+1}^{N} rT_m} \int_0^{T_n} w a_{Ln}\left(t\right) e^{r\left(T_n-t\right)} dt\right)^{\beta_n}, \tag{C.42}$$

where Υ is a constant. Differentiating with respect to r and invoking the envelope theorem, we find that

$$\frac{\partial \ln c_N\left(w,r\right)}{\partial r} = \sum_{n=1}^{N} \beta_n \frac{\int_0^{T_n} \left(\sum\limits_{m=n}^{N} T_m - t\right) w a_{Ln}\left(t\right) e^{\sum\limits_{m=n}^{N} rT_m - rt} dt}{c_N} = \mathcal{APP}_{agg},$$

where $\mathcal{APP}_{agg}$ is an *aggregate* average period of production that takes into account that labor expenditures in upstream stages are further removed from the sale of the final good than labor expenditures in downstream stages.

This aggregate $\mathcal{APP}$ governs the response of the equilibrium wage rate to changes in the interest rate. More specifically, by our choice of numéraire, $c_N = 1$, and we can differentiate equation (C.42) and invoke the envelope theorem to obtain

$$\frac{\partial \ln w}{\partial r} = -\mathcal{APP}_{agg},$$

which is the analog of Proposition 2 and equation (10) in the main text.

This aggregate is not an average of the stage-specific $\mathcal{APP}$s, as in our baseline model (see equation (10)), but rather a weighted sum of these stage-specific $\mathcal{APP}$s, with weights that add up to more than 1. This is most simply illustrated by assuming that labor is only used at the beginning of production, as in Wicksell's lumber model, in which case, we have $\mathcal{APP}_n = T_n$, and

$$\begin{aligned}\mathcal{APP}_{agg} &= \sum_{n=1}^{N} \beta_n \sum_{m=n}^{N} T_m \\ &= T_N + (1-\beta_N)\, T_{N-1} + (1-\beta_N-\beta_{N-1})\, T_{N-2} + ... + \beta_1 T_1.\end{aligned}$$

We can now turn to the equilibrium allocation of labor across stages in a steady-state equilibrium. Given the Cobb-Douglas cost function in equation (C.42), it is straightforward to show that

$$e^{\sum\limits_{m=n+1}^{N} rT_m} \int_0^{T_n} w a_{Ln}(t)\, e^{r(T_n-t)} dt = \beta_n c_N,$$

from which we obtain

$$L_n = \frac{\tilde{\Lambda}_n(r)}{\int_0^1 \tilde{\Lambda}_n(r)\, dn} L,$$

where the term $\tilde{\Lambda}_n(r)$ is given by

$$\tilde{\Lambda}_n(r) = \beta_n \frac{\int_0^{T_v} a_{Ln}(t)\, dt}{\int_0^{T_n} a_{Ln}(t)\, e^{r(T_n-t)} dt} e^{-\sum\limits_{m=n+1}^{N} rT_m}. \tag{C.43}$$

Importantly, this term $\tilde{\Lambda}_n(r)$ remains independent of the wage w given the Cobb-Douglas technology in (C.37). The only difference relative to the analogous equation (8) in our baseline model is the last term, which other things equal, tends to lead to a lower allocation of labor to upstream stages, which are further removed from the collection of revenue from consumers (and thus labor expenditures in those upstream stages accumulate more interest).

We finally consider equilibrium in the capital market. Aggregate capital demand with GVCs is given by

$$K^d = \sum_{n=1}^{N} \int_0^{T_n} w\ell_n(t)\, \frac{e^{\sum\limits_{m=n+1}^{N} rT_m + r(T_n-t)} - 1}{r},$$

with labor expenditures in upstream stages generating higher capital demands. The capital supply is unchanged relative to our baseline model, so the interest rate is pinned down by

$$\frac{K^d}{wL} = \sum_{n=1}^{N} \int_0^{T_n} \frac{\ell_n(t)}{L} \frac{e^{\sum\limits_{m=n+1}^{N} rT_m + r(T_n-t)} - 1}{r} dt = \frac{1}{\rho + m - r} = \frac{K^s}{wL},$$

which is analogous to equation (15) in the main text.

Free Trade Equilibrium: Comparative Advantage We now consider a free trade equilibrium between two countries, Home and Foreign, symmetric in all respects except for the fact that $\phi^H = \rho^H + m^H < \rho^F + m^F = \phi^F$. The main novelty relative to our baseline model is that we allow for trade in the semifinished products associated with the output of the various stages of production, $n = 1, ...N$.

Let us first consider the case of a free trade equilibrium with Home featuring a lower interest rate and a higher wage rate. Remember that we can express the unit cost of production as

$$c_n^j(w, r) = \int_0^{T_n} w^j a_{Ln}^j(t) e^{r^j(T_n - t)} dt + c_{n-1} a_{yn} e^{r^j T_n},$$

and we showed above that

$$\frac{\partial \ln c_n^j(w, r)}{\partial r^j} = \mathcal{APP}_n^j.$$

On this account, the lower interest rate country (Home), will tend to feature disproportionately lower unit costs for the set of goods with longer $\mathcal{APP}$s. We can more generally decompose the cost function as

$$d \ln c_n(w, r) = \mathcal{APP}_n d \ln r^j + \alpha_n d \ln w^j, \tag{C.44}$$

which confirms the result of our baseline model that production costs are shaped by the interaction of relative interest differences across countries and variation in the $\mathcal{APP}$ across sectors. In contrast to our baseline model (see equation (22), in particular) costs are no longer linear in wages, and the effect of wage differences on costs now interacts with the labor intensity of production of a given stage.

From an empirical point of view, this does not pose any difficulties for the results developed in Section 5, since we showed in column (4) of Table 1 that the interaction $\mathcal{APP}_n \times r_{j,t}$ predicts sectoral exports even after controlling for the interaction of GDP-per-capita (a natural proxy for wage differences across countries) and sector fixed effects.

From a theoretical point of view, deriving sharp results of the type in Propositions 3 or 4 is complicated by the fact that wage differences are shaped by the same primitives as differences in interest rates, and by the fact that $\mathcal{APP}$ of a sector is no longer *only* a function of the interest rate r^j in country j. More specifically, the timing of labor and prior-stage input expenditures jointly shape the $\mathcal{APP}$ of a given stage, and the relative use of the two is generally shaped by the relative size of country j's labor and the unit cost of the prior-stage input, which may depend on the other country's wage and interest rate. This breaks the block-recursiveness we had exploited for our baseline model in the main text, where we first solved for equilibria for given interest rates, and then we solved for those interest rates.

Despite these complications, as long as we can rank stages by their ratio $\mathcal{APP}_n/\alpha_n$, and the relative ranking of this ratio for any two stages is not reversed when evaluated at different interest rates *and* wage rates, then a result analogous to Proposition 3 in the main text continues to hold in this version of the model with GVCs. More specifically, we rewrite Definition 3 as:

Definition C.2. Reversals in the ratio $\mathcal{APP}_n/\alpha_n$ do not occur if for any two stages n and n' either (i) $\mathcal{APP}^j_n/\alpha_n < \mathcal{APP}^j_{n'}/\alpha_{n'}$ for all $\left(r^H, r^F, w^H, w^F\right)$ or (ii) $\mathcal{APP}^j_n/\alpha_n > \mathcal{APP}^j_{n'}/\alpha_{n'}$ for all $\left(r^H, r^F, w^H, w^F\right)$.

As long as there are no reversals of this type:

Proposition C.2. As long as there are no reversals of the ratio $\mathcal{APP}_n/\alpha_n$, the country with the lower interest rate necessarily has comparative advantage in stages with relatively high ratios $\mathcal{APP}^j_n/\alpha_n$.

The key for this result is that we can express the ratio of Foreign and Home prices for any two stages n and n' as:

$$\frac{p^F_n}{p^H_n} = \left(\frac{w^F}{w^H}\right)^{\alpha_n} \frac{\gamma^F_n\left(r^F\right)}{\gamma^H_n\left(r^H\right)},$$

where the function $\gamma\left(r^j\right)$ captures the impact of the interest rate on the timing of labor and stage $n-1$ inputs, as delivered by the cost minimization problem in (C.38), and satisfies $d\ln\gamma\left(r^j\right)/dr^j = \mathcal{APP}^j_n$. We can then express

$$\left(\frac{p^F_n/p^H_n}{p^F_{n'}/p^H_{n'}}\right)^{1/\alpha_n} = \frac{\tilde{\gamma}^F_n\left(r^F\right)/\tilde{\gamma}^H_n\left(r^H\right)}{\tilde{\gamma}^F_{n'}\left(r^F\right)/\tilde{\gamma}^H_{n'}\left(r^H\right)}$$

where $\tilde{\gamma}^j_n\left(r^j\right) = \left(\gamma^j_n\left(r^j\right)\right)^{1/\alpha_n}$. Because

$$\frac{\partial\ln\tilde{\gamma}^j_n\left(r^j\right)}{\partial r^j} = \mathcal{APP}^j_n/\alpha_n,$$

we have that $\tilde{\gamma}^j_n\left(r^j\right)$ is log-supermodular in r^j and $\mathcal{APP}^j_n/\alpha_n$. Coupled with the lack of $\mathcal{APP}_n/\alpha_n$, reversals, this ensures that the high interest rate country features disproportionately higher prices in stages with higher ratios $\mathcal{APP}_n/\alpha_n$ (cf., Costinot, 2009a).

In the general equilibrium of the model, wages and interest rates are pinned down by labor-market and capital-market clearing conditions. As mentioned above, the analysis of these conditions is more complex than in our baseline model, but we next sketch how a variant of Proposition 4 continues to hold in (properly defined) *regular* economies, and we later develop a specific example to illustrate the results.

Free Trade Equilibrium: Regular Equilibria Consider first the case with equilibrium factor price differences and $w^H > w^F$ and $r^H < r^F$. In the absence of reversals in the ratio $\mathcal{APP}_n/\alpha_n$, the ratio of relative price p^F_n/p^H_n can only be equal to 1 for at most one stage n^*. To simplify matters, we further assume that the equilibrium is one with complete specialization, that is, in which we can partition the set of stages into two disjoint sets $\mathcal{N}^H$ and $\mathcal{N}^F$ with $\mathcal{N}^j$ being the set of stages in which country $j = H, F$ specializes. Naturally, we have $\mathcal{N}^H \cup \mathcal{N}^F = \{1, 2, 3, ..., N\}$.

Let us next reindex stages in decreasing order of their ratio $\mathcal{APP}_n^j/\alpha_n$. More precisely, when indexing stages by $v = 1, ..., N$, we have $\mathcal{APP}_v^j/\alpha_v$ decreasing in v. Next, as in Dornbusch et al. (1977), define the relative labor efficiency schedule

$$A(v) \equiv \frac{\tilde{\gamma}_v^F\left(r^F\right)}{\tilde{\gamma}_v^H\left(r^H\right)}. \tag{C.45}$$

As long as factor prices vary across countries, with $r^H < r^F$, the function $A(v)$ is necessarily decreasing in v, and there exists a unique $v^* \in (0, 1)$, such that $A(v) > w^H/w^F$ for $v < v^*$ and $A(v) < w^H/w^F$ for $v > v^*$. In words, the low interest rate Home has a relative cost advantage in goods v with $v < v^*$, while Foreign has a relative cost advantage in all stages with $v > v^*$.

Next, consider labor demand and labor-market clearing. Following analogous derivations as those in the closed economy, we find

$$\frac{w^H}{w^F} = \frac{\sum\limits_{v<v^*} \tilde{\Lambda}_v^H\left(r^H\right)dv}{\sum\limits_{v>v^*} \Lambda_v^F\left(r^F\right)dv}\frac{L^F}{L^H} \equiv B(v^*), \tag{C.46}$$

where $\tilde{\Lambda}_v^j$ is defined in equation (C.43). The schedule $B(v^*)$ is an increasing function of the threshold stage v^* because as more stages are performed at Home, the numerator in (26) increases while the denominator decreases. Furthermore, when $v^* \to 0$, $B(v^*) \to 0$, and when $v^* \to 1$, $B(v^*) \to \infty$. In sum, as in our baseline model, equations (C.45) and (C.46) determine the unique equilibrium threshold v^* and relative wage w^H/w^F for given interest rates $r^H < r^F$.

To solve for the equilibrium interest rates, we turn to a discussion of capital demand. Following the same derivations as in the closed-economy version of our model, and invoking the Cobb-Douglas structure in (C.37), we have that sectoral capital demand is given by

$$K_v^j = \int_0^{T_v} \frac{1}{\alpha_v} w^j \ell_v^j(t) \frac{e^{r^j(T_v-t)}-1}{r^j}dt. \tag{C.47}$$

The Home and Foreign interest rates are then pinned down by

$$\sum_{v=1}^{N} \frac{L_v^H}{L^H} \times \frac{K_v^H}{L_v^H}dv = \frac{w^H}{\phi^H - r^H} \tag{C.48}$$

and

$$\sum_{v=1}^{N} \frac{L_v^F}{L^F} \times \frac{K_v^F}{L_v^F} dv = \frac{w^F}{\phi^F - r^F}, \tag{C.49}$$

respectively.

As long as these equations deliver a lower interest rate at Home than in Foreign, the pattern of specialization will indeed be as described above, with Home fully specializing in the stages with the highest values $\mathcal{APP}_n^j/\alpha_n$ $(v < v^*)$. The fact that Home is more patient or more financially developed than Foreign (i.e., $\phi^H < \phi^F$) tends to make this equilibrium more likely, but whenever stages with higher ratios $\mathcal{APP}_v^j/\alpha_v$ also turn out to be more capital intensive, the integrals in the left-hand-side of the above expressions – which are employment-weighted average capital intensities – will *tend to* be higher for Home and than for Foreign, which constitutes a force towards a higher interest rate at Home. To formalize such a link, we follow the baseline model and define the existence of *modified* time-versus-capital intensity ranking reversals as follows:

Definition C.3. *A modified time-versus-capital intensity ranking reversal occurs whenever for two stages v and v', $\mathcal{APP}_v/\alpha_v < \mathcal{APP}_{v'}/\alpha_{v'}$ but $K_v^j(w^j, r)/L_v^j(w^j, r) > K_{v'}^j(w^j, r)/L_{v'}^j(w^j, r)$, for some w and r.*

Given the similarities between the properties of this variant of the model with GVCs and our baseline model, it is then clear that the following variant of Proposition 4 will hold:

Proposition C.3. *In the absence of reversals in the ratio $\mathcal{APP}_v/\alpha_v$ and of modified time-versus-capital intensity ranking reversals, and provided that capital intensity K_v^j/L_v^j is decreasing in the interest rate, a world equilibrium is one of two types:*

a) *A complete-specialization equilibrium in which the country with the higher aggregate capital-labor ratio (K^s/L) features a lower interest rate and completely specializes in stages with relatively high ratios $(\mathcal{APP}/\alpha)$; or*

b) *An equilibrium featuring factor price equalization and an indeterminate pattern of specialization.*

Furthermore, when aggregate capital-labor ratios are sufficiently similar in the two countries, the equilibrium is necessarily one with factor price equalization.

This result can be proven following the same exact steps as in the proof of Proposition 4 in Appendix A.4, so we omit the proof here.

As in our baseline model, in the presence of modified time-versus-capital intensity ranking reversals, there is the possibility that, for given parameter values (patience rates, monitoring costs, population sizes, etc.), there might exist *multiple equilibria* with qualitatively different patterns of specialization in different equilibria, some of which delivering predictions at odds with Proposition C.3 as well as the Heckscher-Ohlin model. Because the results are analogous to those in our baseline model, and because their formal derivation is analogous to our analysis in Online Appendix C.4, we omit the details here. We next, however, illustrate these results via a specific example.

An Illustration: the Wicksell-Swan Timber Model with Sequential Stages Consider the special case in which stage n output is given by

$$y_n(L_n, y_{n-1}) = (Z_n(T_n)\ell_n(0))^{\alpha_n} (y_{n-1}(0)))^{1-\alpha_n}, \tag{C.50}$$

with $\alpha_n \in (0,1)$ and $\alpha_1 = 1$. In words, we adopt Wicksell's point-input, point-output formulation in which labor and the prior stage input are only employed at the very beginning of production, and all that firms need to do is to wait for these to mature for an interval T_n. For simplicity, we also assume that the function $Z_n(T_n)$ is a constant-elasticity function with $Z_n(T_n) = T_n^{\zeta_n}$, as in our Wicksell-Swan model with horizontal sectors. Naturally, the $\mathcal{APP}$ of each stage n is given by $\mathcal{APP}_n = T_n$, and profit-maximization delivers $T_n = \alpha_n\zeta_n/r^j$ (see Antràs, 2023a). Because the ratio $\mathcal{APP}_n/\alpha_n$ is given by $\mathcal{APP}_n/\alpha_n = \zeta_n/r^j$, it follows immediately that there is no possibility for reversals in the ratio $\mathcal{APP}_n/\alpha_n$, as defined in Definition C.2. Furthermore, comparative advantage is shaped *purely* by cross-sectoral variation in the time intensity parameters $\boldsymbol{\zeta} = \{\zeta_1, \zeta_2, ..., \zeta_v\}$.

Furthermore, as we show in Antràs (2023a), the relative cost at which Home can produce stage n relative to Foreign is given by:

$$\frac{c_n^H\left(w^H, r^H, p_{n-1}\right)}{c_n^F\left(w^F, r^F, p_{n-1}\right)} = \left(\frac{w^H}{w^F}\left(\frac{r^H}{r^F}\right)^{\zeta_n}\right)^{\alpha_n},$$

and thus Home features a lower cost for all stages in which $w^H/w^F < A(n) = \left(r^F/r^H\right)^{\zeta_n}$. With free trade, we can reindex stages in decreasing order of their time intensity ζ_n, so we have ζ_v decreasing in v. In such a case, and the function $A(v)$ is necessarily decreasing in v, and Home has comparative advantage in stages with relatively higher time intensity.

Next, consider labor demand and labor-market clearing. Solving for $\tilde{\Lambda}_v(r)$ in (C.43) while invoking $r^j T_v^j = \alpha_v \zeta_v$, and plugging into (C.46), delivers

$$\frac{w^H}{w^F} = \frac{\sum\limits_{v<v^*} \beta_v e^{-\sum\limits_{m=v}^{N} \alpha_m \zeta_m}}{\sum\limits_{v>v^*} \beta_v e^{-\sum\limits_{m=v}^{N} \alpha_m \zeta_m}} \equiv B\left(v^*\right).$$

The schedule $B\left(v^*\right)$ is an increasing function of the threshold stage v^* because as more stages are performed at Home, the numerator in (26) increases while the denominator decreases.

For given interest rates $r^H < r^F$, equations (25) and (26) determine the equilibrium threshold v^* and the relative wage w^H/w^F in a manner completely analogous to the analysis in our baseline model and as in Dornbusch et al. (1977).

We can next express capital intensity at stage v in country j as

$$\frac{K_v^j}{L_v^j} = \frac{w^j}{r^j} \frac{1}{\alpha_v} \left(e^{\alpha_v \zeta_v} - 1\right). \tag{C.51}$$

Finally, the equations pinning down the interest rates r^H and r^F can be obtained by plugging (C.43) and (C.51) into (C.48) and (C.49).

It is straightforward to see that equation (C.51) implies that (i) capital intensity necessarily decreases in the interest rate r^j, and (ii) there cannot be capital intensity reversals.

Nevertheless, for certain values of the vector $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, ..., \alpha_v\}$, there can certainly exist modified time-versus-capital intensity ranking reversals, in the sense that K_v^j/L_v^j need not always be higher for stages with a lower index v (i.e., lower values of ζ_v). There is of course a close resemblance between the analysis of this Wicksell-Swan model with many stages and our analysis of the variant of the model with labor used to chop off trees.

The consequences of the possibility of modified time-versus-capital intensity ranking reversals are analogous to those in the main text. For sufficiently small differences in patience across countries, there arises the possibility of multiple equilibria, with Foreign specializing in low-v stages in one of those equilibria (provided that those stages happen to feature low capital intensity). Naturally, in those situations, there remains an equilibrium in which it is Home that specializes in these low capital-intensity stages, thereby putting further downward pressure on their interest rate relative to the Foreign one. Finally, close to the symmetric situation with $\phi^H = \phi^F$, there remains the possibility of an equilibrium with factor price equalization, though in some cases, that equilibria can coexist with the equilibria with complete specialization described above. The intuition behind these results is completely analogous to that developed to explain our similar results in our model with 'horizontal' sectors.

C.7 Costly-Trade Equilibrium in GVCs

In this Appendix, we relax the assumption that international trade does not entail trade costs in our extension of the model with GVCs. We first consider the typical formulation of trade costs as being 'instantaneous' iceberg costs proportional to the value of the good being transacted, and show that these instantaneous trade costs generate nontrivial effects on the pattern of comparative advantage, in line with insights from the work of Harms et al. (2012), Baldwin and Venables (2013), and Antràs and de Gortari (2020). We next leverage the explicit treatment of time in our model to study the effect of trade costs associated with the time it takes to ship goods across borders, and show how world interest rates shape the pattern of specialization.

To simplify matters, throughout this section, we develop our results for the special case of the Wicksell-Swan timber model with sequential stages, as developed at the end of the previous section.

Standard Iceberg Trade Costs

As in our baseline model, we begin by assuming that in shipping between Home and Foreign a fraction of goods melts in transit, so τ units of a good need to be shipped for 1 unit to make it to the other country. If one were to ignore the sequentiality of production, treating each stage as an independent production process, we would recover the results in Section 6.1 of the main text, applying to a model with horizontally differentiated sectors. More specifically, if we denote by c_n^j the cost of production of 'good' n in country j, Home would export good n to Foreign only if $\tau c_n^H < c_n^F$, while Foreign would export that good to Home only if $c_n^H > \tau c_n^F$. As a result, a range of goods featuring small production cost differences across countries would become nontraded. More precisely, goods n satisfying

$$\tau \geqslant \max \left\{ \frac{c_n^H}{c_n^F}, \frac{c_n^F}{c_n^H} \right\}$$

would not be traded. Naturally, the lower are trade costs, the larger would be the set of goods traded and the larger the volume of international trade.

In the current version of the model with sequential production, the effects of introducing trade costs are much more complex. First of all, and as formally shown in Antràs and de Gortari (2020), with trade costs it is no longer possible to determine comparative advantage stage by stage. More specifically, the choice at stage n between purchasing stage $n-1$ from Home or Foreign is not independent of where producers at stage $n-1$ source the stage $n-2$ input. Determining the cost minimization production path to sell final goods in a given country (Home or Foreign) involves solving an optimal *path* problem. More formally, iterating the cost function dual to the stage production functions in equation (37), we can write the problem of a *lead firm* choosing the location of production of all stages $n \in N$ to minimize the overall cost of serving consumers in a given country $i \in \{H, F\}$ as

$$\boldsymbol{\ell}^i = \arg\min_{\boldsymbol{\ell} \in 2^N} \left\{ \prod_{n=1}^{N} \left(c_n^{\ell(n)}\right)^{\beta_n} \times \prod_{n=1}^{N-1} \left(\tau_{\ell(n)\ell(n+1)}\right)^{\beta_n/\alpha_n} \times \tau_{\ell(N)i} \right\}, \tag{C.52}$$

where $c_n^{\ell(n)}$ is the cost of production (inclusive of interest payments) of stage n in location $\ell(n)$, where β_n was defined in equation (C.41), and where $\tau_{\ell(n)\ell(n+1)} = \tau$ when $\ell(n) \neq \ell(n+1)$ and $\tau_{\ell(n)\ell(n+1)} = 1$ when $\ell(n) = \ell(n+1)$.

In the absence of trade costs, the problem above reduces to minimizing production costs stage by stage, or choosing $\ell^i(n) = \arg\min \left\{c_n^{\ell(n)}\right\}$ regardless of the destination i of final consumption. With trade costs, however, the location $\ell^i(n)$ minimizing $c_n^{\ell(n)}$ might not be part of a firm's optimal path. More specifically, and as emphasized by Harms et al. (2012) and Baldwin and Venables (2013), the presence of trade costs may lead to the *bunching* (or co-location) of two or more contiguous production stages in the same location, even if some of those stages could be produced more cheaply in the other country. Intuitively, facing a trade-off between selecting the cheapest location for an individual step and incurring additional transport costs, firms may tend to lump together several parts of the production chain in one country, even if this implies that some steps are not performed at the cheapest location in terms of production costs. Harms et al. (2012) and Baldwin and Venables (2013) develop very stylized examples that illustrate the rich comparative advantage patterns that emerge depending on different levels of trade costs.[45]

We next offer an alternative formal derivation of the benefits of bunching contiguous production stages in the presence of trade costs. To do so, we build on a dynamic programming formulation of the path problem (C.52). As Antràs and de Gortari (2020) and Tyazhelnikov (2022) show, as long as technologies feature constant returns to scale, the *lead firm* can break the problem into a series of stage- and country-specific optimal sourcing problems, and then solve the problem via forward induction (starting in the most upstream stage).

More specifically, consider the (sub)problem of a producer at any stage $n+1$ choosing the optimal source of stage n inputs. In our GVC model with free trade, this producer would simply choose a supplier at Home if $w^F \left(r^F\right)^{\zeta_n} > w^H \left(r^H\right)^{\zeta_n}$ and a supplier in Foreign if $w^F \left(r^F\right)^{\zeta_n} < w^H \left(r^H\right)^{\zeta_n}$. In the presence of iceberg trade costs, this choice will obviously be a function of trade barriers, but one also needs to take into account whether these trade barriers might affect the cost of intermediate inputs as well as the optimal length of production, and

[45]A second noteworthy aspect of the minimand in equation (C.52), also pointed out by Antràs and de Gortari (2020), is that the trade-cost elasticity of the unit cost of serving consumers in country j increases along the value chain due to the compounding effect of trade costs along the chain. An implication of this compounding effect, in a multi-country environment, is that relatively central countries gain comparative advantage in relatively downstream stages of production.

thus labor productivity and production costs.

Our first result, analogous to similar results obtained in other variants of our model, is that the level of (iceberg) trade costs has *no* bearing on the length of the production process at any stage. To see this, consider the problem of producers at stage n in country $j \in \{H, F\}$, who set the time length t_n^j to maximize their profits,

$$\pi_n^j = \sum_{j' \in \{H,F\}} \left[p_n^{jj'} \left(Z_n \left(T_n^j \right) L_n^{jj'} \right)^{\alpha_n} \left(y_{n-1}^{jj'} \right)^{1-\alpha_n} e^{-r^j T_n^j} - w^j L_n^{jj'} - p_{n-1}^j y_{n-1}^{jj'} \right], \tag{C.53}$$

which are the sum of revenue minus costs associated with sales in each market. Regardless of trade costs, which shape the wedge between domestic (p_n^{jj}) and export prices ($p_n^{jj'}$), the optimal production length will satisfy

$$\alpha_n \frac{Z_n' \left(T_n^j \right)}{Z_n \left(T_n^j \right)} = r^j,$$

just as in the case without trade costs. The intuition for this result was anticipated in our closed-economy analysis in Section 2. Although the marginal benefit of letting production mature increases proportionately with the scale of production, the marginal cost of delaying the collection of revenue also grows proportionately with that scale, as wage payments are also proportional to scale.

When the function $Z_n(t)$ is log-linear, as in Swan's version of the model, it follows from the previous discussion that the cost of production for a producer from country j at stage n is given by

$$c_n^j = \kappa_n \left(\frac{w^j \left(r^j \right)^{\zeta_n}}{\left(\alpha_n \zeta_n \right)^{\zeta_n}} \right)^{\alpha_n} \left(p_{n-1}^j \right)^{1-\alpha_n} e^{\alpha_n \zeta_n}, \tag{C.54}$$

for some constant κ_n. Thus, a producer in Foreign at stage $n+1$ would choose a Foreign supplier at n whenever

$$\tau \left(w^H \left(r^H \right)^{\zeta_n} \right)^{\alpha_n} \left(p_{n-1}^H \right)^{1-\alpha_n} > \left(w^F \left(r^F \right)^{\zeta_n} \right)^{\alpha_n} \left(p_{n-1}^F \right)^{1-\alpha_n}. \tag{C.55}$$

This expression differs from the free-trade comparison of $w^H \left(r^H \right)^{\zeta_n}$ and $w^F \left(r^F \right)^{\zeta_n}$ in two respects. First, the left-hand-side includes the iceberg trade costs associated with the Foreign buyer at $n+1$ importing inputs from Home. Naturally, this force produces a direct benefit of co-locating stages $n+1$ and n in the same country (Foreign in this case). A second new aspect of the above inequality (C.55) is that the price paid for inputs at $n-1$ may also differ depending on the choice of location of stage n inputs, due again to trade costs. As a result, the producer at $n+1$ in Foreign might favor sourcing from Home at n if the price of $n-1$ inputs is significantly lower for Home producers at n than for Foreign producers at n, perhaps because

Home has strong cost advantage at stage $n-1$. Notice that this possibility would also lead to some 'bunching' at Home of stages n and $n-1$. But leaving this aside, we next show that a producer at $n+1$ in Foreign will *necessarily* be more likely to choose a supplier from Foreign at n than as implied by a simple comparison of $w^H \left(r^H\right)^{\zeta_n}$ and $w^F \left(r^F\right)^{\zeta_n}$ in the absence of trade costs.

To show this formally, consider the prices p^H_{n-1} and p^F_{n-1} that partly shape the choice at n, as indicated by inequality (C.55). Given our discussion above, it should be clear that these prices satisfy:

$$\begin{aligned} p^H_{n-1} &= \min\left\{\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}, \tau\left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}\right\}; \\ p^F_{n-1} &= \min\left\{\tau\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}, \left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}\right\}, \end{aligned}$$

where p^H_{n-2} is the price of stage $n-2$ faced by producers in H of stage $n-1$, and p^F_{n-2} is the analogous price for producers in F of stage $n-1$ output. It is then relatively straightforward to show that the ratio p^H_{n-1}/p^F_{n-1} satisfies

$$\frac{p^H_{n-1}}{p^F_{n-1}} \geqslant \frac{1}{\tau},$$

which can be interpreted as a no-arbitrage condition in the sourcing of inputs at stage $n-1$.[46] Furthermore, this inequality is strict unless both H and F producers source from H at $n-2$. This inequality then implies that a sufficient but *not necessary* condition for (C.55) to hold is

$$w^H \left(r^H\right)^{\zeta_n} > w^F \left(r^F\right)^{\zeta_n}.$$

In other words, Foreign producers at stage $n+1$ may choose F as a source of stage n inputs even when $w^F \left(r^F\right)^{\zeta_n} > w^H \left(r^H\right)^{\zeta_n}$, which indicates a disproportionate desire to bunch contiguous stages in the same location.

This disproportionate desire to source inputs from a local supplier applies to all stages

[46]The ratio p^H_{n-1}/p^F_{n-1} can thus be bounded below as follows:

$$\frac{p^H_{n-1}}{p^F_{n-1}} = \begin{cases} \dfrac{\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}}{\tau\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}} = \dfrac{1}{\tau} \\ \dfrac{\tau\left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}}{\left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}} = \tau \\ \dfrac{\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}}{\left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}} > \dfrac{\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}}{\tau\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}} = \dfrac{1}{\tau} \\ \dfrac{\tau\left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}}{\tau\left(w^H \left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^H_{n-2}\right)^{1-\alpha_{n-1}}} > \dfrac{\tau\left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}}{\left(w^F \left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} \left(p^F_{n-2}\right)^{1-\alpha_{n-1}}} = \tau \end{cases},$$

where we have appealed to the min operator to insert the inequality in the third and fourth values.

$n > 1$. But it also applies to the case $n = 1$, in which case, the condition in (C.55) reduces to $\tau w^H \left(r^H\right)^{\zeta_1} > w^F \left(r^F\right)^{\zeta_1}$, which clearly also favors local sourcing. Following a completely analogous set of steps (exploiting $\tau \geqslant p^H_{n-1}/p^F_{n-1}$), one can similarly demonstrate that Home suppliers at $n+1$ are also disproportionately more likely to choose Home suppliers at n than as implied by the simple condition $w^H \left(r^H\right)^{\zeta_n} < w^F \left(r^F\right)^{\zeta_n}$ emanating from our baseline model with free trade.

The above discussion shows that, for given factor prices, in a costly-trade equilibrium, producers have a disproportionate incentive to use local versus foreign intermediate inputs in their production, which translates into lower levels of foreign value added in exports. Conversely, a decline of trade costs leads to an 'unbundling' of production stages and a higher ratio of foreign value added in exports. An important caveat, however, is that this result is partial equilibrium in nature. In general equilibrium, changes in trade costs will not only lead to more 'home bias' in sourcing, but it will also affect factor prices. The manner in which factor prices change when trade costs increase depends in subtle ways on the pairwise correlations between upstreamness, time intensity, and capital intensity. The equations characterizing this general equilibrium with costly trade are developed in the Online Appendix of Antràs (2023a), where we also discuss under which circumstances these general equilibrium effects are likely to be modest.

Time as a Trade Cost

In this section, we introduce a temporal dimension of trade costs. Specifically, we now assume that, relative to domestic transactions, shipping goods across borders involves an additional interval of time d between the time at which production is completed and the time at which the shipment is received and payment is made. We also continue to assume that shipping involves iceberg trade costs τ (shipping fees, insurance, etc.) incurred after production but before the shipment arrives at the importer's country.

As in the case of iceberg trade costs, the presence of temporal trade costs turns out to be immaterial for the length of production processes.[47] As a result, the cost of production c^j_n for producers at stage n in country j is still given by (C.54), while the minimum price at which

[47] In particular, going back to equation (C.53), the production length T^j_n that maximizes

$$\pi^j_n = \sum_{j'=\{H,F\}} \left[p^{jj'}_n \left(Z_n \left(T^j_n\right) L^{jj'}_n \right)^{\alpha_n} \left(y^{jj'}_{n-1} \right)^{1-\alpha_n} e^{-r^j \left(T^j_n + d^{jj'}\right)} - w^j L^{jj'}_n - p^j_{n-1} y^{jj'}_{n-1} \right]$$

is still given by

$$\alpha_n \frac{Z'_n \left(t^j_n\right)}{Z_n \left(t^j_n\right)} = r^j,$$

regardless of the shipping time $d^{jj'} = d$ if $j \neq j'$ (and $d^{jj'} = 0$ if $j = j'$).

these goods can be sold in country j' is

$$p_n^{jj'} = \tau^{\mathcal{I}(j \neq j')} e^{r^j d^{\mathcal{I}(j \neq j')}} c_n^j = \begin{cases} \tau e^{r^j d} c_n^j & \text{if } j' \neq j \\ c_n^j & \text{if } j' = j \end{cases} .$$

Some of the implications of these temporal trade costs are analogous to those resulting from iceberg trade costs. Naturally, shipping delays are relatively more costly for downstream stages than for upstream stages (as in Antràs and de Gortari, 2020), and high shipping times lead to more bunching (or co-location of contiguous stages) than implied by the model without these international trade costs. Nevertheless, the above modeling of trade costs as an interval of time between production and delivery generates at least two additional implications.

The first is analogous to the one derived in our baseline model with horizontal sectors. Because with trade costs the equilibrium is generically one without factor price equalization, we necessarily have that either $\tilde{\tau}^H < \tilde{\tau}^F$ or $\tilde{\tau}^H > \tilde{\tau}^F$ (where $\tilde{\tau}^j \equiv \tau e^{r^j d}$) and thus trade costs are asymmetric and relatively lower in the country featuring a lower interest rate. As mentioned in the main text, this implication is in line with the results in Waugh (2010), who shows that to reconcile bilateral trade volumes and price data within a standard gravity model, trade frictions between rich and poor countries must be systematically asymmetric, with poor countries facing higher costs to export relative to rich countries.

Second, because trade costs are now partly shaped by the time value of money, reductions in interest rates worldwide tend to reduce trade costs. From our results above on the link between iceberg trade costs and GVC participation, we can thus conclude that proportional reductions in interest rates that leave relative wages unchanged (see the discussion in the Online Appendix of Antràs, 2023a) will necessarily lead to an increase in the extent to which goods cross borders, hence generating a higher volume of GVC participation. As a result, and as mentioned in the main text, one potential contributor to the growth in GVC activity in the late 1980s, 1990s and 2000s might have been the secular decline in interest rates during that period.